Upstate New York Online Number Theory Colloquium

Time and Date: 1:30 pm EST November 8, 2021

Speaker: Dino Lorenzini

Title: Torsion and Tamagawa numbers

Abstract: Associated with an abelian variety A/K over a number field K is a finite set of integers greater than 1 called the local Tamagawa numbers of A/K. Assuming that the abelian variety A/K has a K-rational torsion point of prime order N, we can ask whether it is possible for none of the local Tamagawa numbers to be divisible by N. The ratio (product of the Tamagawa numbers)/|Torsion in $E(K)$| appears in the conjectural leading term of the L-function of A in the Birch and Swinnerton-Dyer conjecture, and we are thus interested in understanding whether there are often cancellation in this ratio.

We will present some finiteness results on this question in the case of elliptic curves. More precisely, let $d > 0$ be an integer, and assume that there exist infinitely many fields $K/\mathbb{Q}$ of degree d with an elliptic curve E/K having a K-rational point of order N. We will show that for certain such pairs (d, N), there are only finitely many fields $K/\mathbb{Q}$ of degree d such that there exists an elliptic curve E/K having a K-rational point of order N and none of the local Tamagawa numbers are divisible by N. The lists of known exceptions are surprisingly small when d is at most 7.