Calculus 3 Final Sample Exam 3

courtesy of Dr. Inna Sysoeva, University of Pittsburgh, adapted

Problem 1. a) Find the unit tangent vector $\mathbf{T}$ and unit normal vector $\mathbf{N}$ to the curve $\mathbf{r}(t) =< 3\cos t, 4t, 3\sin t >$ at the point $P = \left(-\frac{3}{\sqrt{2}}, 3\pi, \frac{3}{\sqrt{2}}\right)$.

b) Find the curvature of the curve at the point P.

Problem 2. Use linear approximation to approximate the number $\sqrt{3.04 + e^{-0.08}}$.

Problem 3. Determine all local maxima, local minima and saddle points of $f(x, y) = 3y - y^3 - 3x^2y$.

Problem 4. A rectangular box without a lid is to be made from 48 ft^2 of cardboard. Find the maximum volume of the box.

Problem 5. Find the $y-$ coordinate of the center of mass of a lamina that occupies the region bounded by $y^2 = x + 4,\ x = 0$, and $y \geq 0$ and has density $\rho(x, y) = y$. Simplify your answer as much as possible.

Note: The $y-$ coordinate of the center of mass of a lamina with density function ρ is $(\iint\limits_D y\rho\, dA)/(\iint\limits_D \rho\, dA)$.

Problem 6. Find the volume of the solid that lies within the cylinder $x^2 + y^2 = 4$, above the $(x, y)-$plane, and below the cone $z^2 = 4x^2 + 4y^2$.

Problem 7. Let $\mathbf{F}$ be the two-dimensional vector field given by $\mathbf{F}(x, y) =< ye^{xy} - 1, xe^{xy} + 2y >$.

a) Determine if $\mathbf{F}$ is a conservative vector field, and if so, find a potential function.

b) Find the value of the line integral $\int\limits_C \mathbf{F}\cdot\mathbf{T}ds$, where C is the line segment from $(0, 3)$ to $(5, 0)$.

Problem 8. Use Green's Theorem to find the value of $\oint\limits_C -5x^2\, dx+7xy\, dy$, where C is the closed curve consisting of the edges of the triangle with vertices $(0, 0),\ (3, 1)$, and $(0, 3)$, oriented counterclockwise.

Problem 9. Use Divergence Theorem to find the total flux $\iint\limits_S \mathbf{F}\cdot\mathbf{n}\, dS$ of the vector field $\mathbf{F}(x, y, z) =< x^2, yz^2, -2xz >$ across the surface S given by $x^2 + y^2 + z^2 = 2$ with outward orientation.

Problem 10. Use Stokes' theorem to evaluate $\oint\limits_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = e^x\mathbf{i} + (x^2 + y^2)\mathbf{j} + z\mathbf{k}$, and C is the boundary of the part of the plane $2x + y + 2z = 2$ in the first octant oriented counterclockwise when viewed from above.