2008 A.B.

Quotient singularities,
Integer Ratios of Factors
and RH

Ex. $X \subseteq T^4$

1) X is closed
2) $\forall n \in \mathbb{N}$ $\forall x \in X$
$nx \in X$

Then X is a finite
union of closed subgroups
by Lawrence (1980)

Ex. If instead of T^4
we consider $SO_4(\mathbb{R})$

$X_0 = \{A \in SO_n(\mathbb{R}) \mid A^2 = I\}$

Any closed subset X of X_0, containing I, has the property that

$\forall x \in X$, $\forall n \in \mathbb{N}$ $\quad nx \subset X$

Riemann Hypothesis

Nyman (1950) - Beurling (196?) - Baez-Duarte Criterion (2003)

$f_2 = \{1, 0, 1, 0, 1, 0, \ldots\}$

$f_3 = \{1, 2, 0, 1, 2, 0, \ldots\}$

$f_4 = \{1, 2, 3, 0, 1, 2, 3, 0, \ldots\}$

$\ldots$

$L_2 = \{a_n\} \qquad |a_n|^2 = \sum \frac{|a_n|^2}{n^2}$

$$RH \iff \mathbb{1} \in \overline{\operatorname{span}_{\mathbb{R}}(f_2, f_3, \dots)}$$

$$f_k(n) = k\left\{\frac{n}{k}\right\}$$

$(9, 1, -2, -3, -5)$

one of 29 "stable quintiples" of Mori, Morrison, Morrison

$\forall p \geq 11$

$\frac{1}{p}(9, 1, -2, -3)$

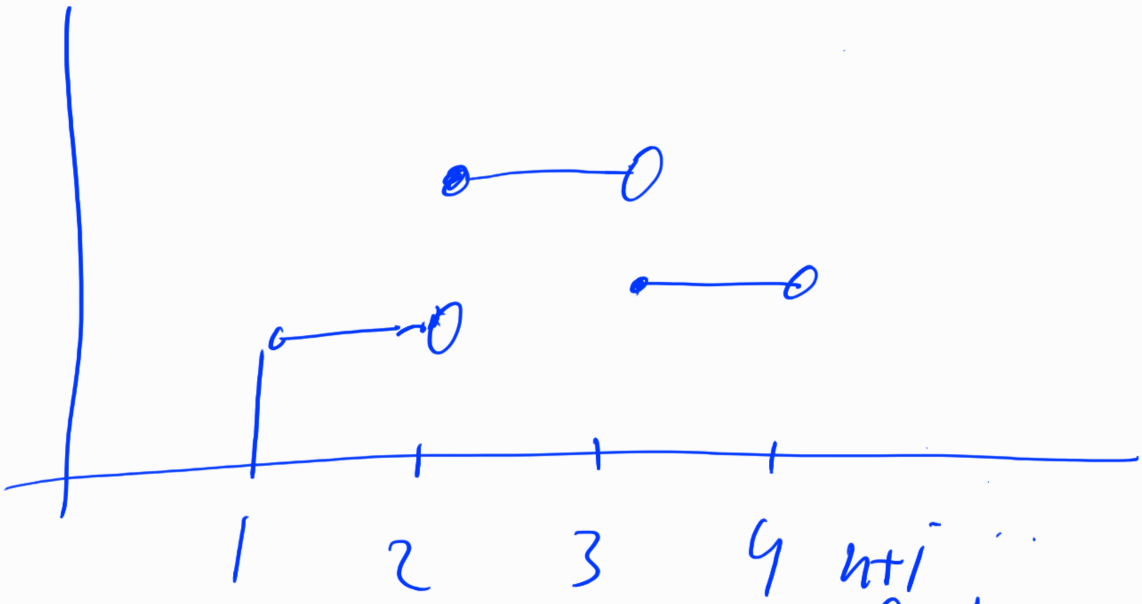
...

$\frac{1}{p}(1, -2, -3, -5)$

terminal

$$\left\{\frac{9k}{p}\right\} + \left\{\frac{k}{p}\right\} + \left\{\frac{-2k}{p}\right\} + \left\{\frac{-3k}{p}\right\} + \left\{\frac{-5k}{p}\right\} \in \{2, 3\}$$

Vasyunin (1999)



$$\int_n^{n+1} \frac{1}{x^2}\,dx = \frac{1}{n(n+1)}$$

$$\int_1^\infty \frac{\left(f(x)\right)^2}{x^2}\,dx$$

$$\sum_{n=1}^\infty \frac{\left(f(n)\right)^2}{n(n+1)}$$

$$T_m f(x) = f\left(\frac{x}{m}\right)$$

$$f_k(x) = \left[\frac{x}{k}\right] - \frac{1}{k}[x]$$

$$f_k(x) = -\left\{\frac{x}{k}\right\} + \underline{\frac{1}{k}\{x\}}$$

$n \in \mathbb{N}$

$$f_k(n) = -\left\{\frac{n}{k}\right\}$$

$$\sum_{\substack{k \in K \\ \text{finite}}} c_k \left[\frac{x}{k}\right] \quad \Big| \quad \sum c_k = 0$$

He looked for linear combinations of step-functions like this that take values in $\{0\}, \{1\}$.

$$\left[\frac{x}{n}\right] - \left[\frac{x}{n+1}\right] - \left[\frac{x}{n(n+1)}\right]$$

$k = 1, 2, 3, \ldots$

k positive, $k+1$ negative terms.

$k = 2$ 29   step-functions

$k = 3$ 21

$k = 4$ 2

$9, 1, -2, -3, -5$

$$\{9x\} + \{x\} + \{-2x\} + \{-3x\} + \{-5x\} \in \{2, 3\}$$

$\forall x \notin \mathbb{Z}$

$$\{9x\} + \{x\} - \{2x\} - \{3x\} - \{5x\} \in \{-1, 0\}$$

$$\lfloor 9x \rfloor + \lfloor x \rfloor - \lfloor 2x \rfloor - \lfloor 3x \rfloor - \lfloor 5x \rfloor \in \{0, 1\}$$

$$\operatorname{lcm}(9, 1, 2, 3, 5) = 90$$

$$\left\lfloor \frac{x}{10} \right\rfloor + \left\lfloor \frac{x}{90} \right\rfloor - \left\lfloor \frac{x}{45} \right\rfloor - \left\lfloor \frac{x}{30} \right\rfloor - \left\lfloor \frac{x}{18} \right\rfloor \in \{0, 1\}$$

Integer Ratios of Factorials

$$\frac{(n+m)!}{n!\,m!} \in \mathbb{Z}$$

$$\frac{(2n)!\,(2m)!}{n!\,m!\,(n+m)!} \in \mathbb{Z}$$

p prime

order of p in $n!$

is $\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p^3}\right]+\ldots$

$$[2x]+[2y]-[x]-[y]-[x+y]\geq 0$$

$$\in\{0,1\}$$

Landau, 1900

Picon ...

$$\frac{(9n)!\,n!}{(2n)!\,(3n)!\,(5n)!}\in\mathbb{Z}$$

Fernando Rodriguez Villegas

$$\sum_{n=0}^{\infty}\frac{(9n)!\,n!}{(2n)!\,(3n)!\,(5n)!}x^n$$

is an <u>algebraic</u> function

Beukers - Heckmann 1989

Bober (2009)

29

21

2

All lattice-free simplexes

in dim 4

Francisco Santos,

Oscar Iglesias-Valiño 2018

Bober-Bell

Bombieri-Bourgain

Soundararajan ...

$1 + 2^{-s} + 3^{-s} + \ldots$

every k-th term,

subtract k $\cdot$ that.