Chapter 1

1. What are all the solutions to the system $x - 2y = 3$ and $-2x + 4y = -6$?

   (a) $x = 1$ and $y = -2$, or $x = -2$ and $y = 4$

   (b) $x - 2y = 3$ and $0 = 0$

   (c) $x = 1$ and $y = -1$

   (d) $x = 3$ and $y = -6$

   (e) x and y are both 0

   (f) x can be arbitrary, and $y = \frac{x-3}{2}$

   (g) There are no solutions

2. What are all the solutions to the system $x + 2y = 3$ and $3x + y = 4$?

   (a) $x = 1$ and $y = 2$, or $x = 3$ and $y = 1$

   (b) $x = 3$ and $y = 4$

   (c) $x = 1$ and $y = 1$

   (d) x can be arbitrary, and $y = 4 - 3x$

   (e) x can be arbitrary, and $y = \frac{3-x}{2}$

   (f) There are infinitely many solutions

   (g) There are no solutions

3. What are all the solutions to the system $x + 2y = 3$ and $2x + 4y = 5$?

   (a) $x = 0$ and $y = 1$

   (b) $0 = -1$

   (c) x can be arbitrary, and $y = \frac{5-2x}{4}$

   (d) x can be arbitrary, and $y = \frac{3-x}{2}$

   (e) This question cannot be answered correctly

   (f) There are infinitely many solutions

   (g) There are no solutions

4. If one is solving three linear equations involving two unknowns, what happens?

   (a) There will always be infinitely many solutions
   (b) There will always be a solution
   (c) Usually there will be no solution, but occasionally there will be one or more solutions
   (d) Usually there will be one solution, but occasionally there will be no solutions or infinitely many solutions
   (e) Usually there will be infinitely many solutions, but occasionally there will be one or no solutions
   (f) There will always be exactly one solution
   (g) There will never be a solution
   (h) Anything can happen

5. If one is solving two linear equations involving three unknowns, what happens?

   (a) Usually there will be infinitely many solutions, but occasionally there will be no solutions
   (b) There will never be a solution
   (c) Usually there will be one solution, but occasionally there will be no solutions or infinitely many solutions
   (d) There will always be a solution
   (e) There will always be exactly one solution
   (f) There will always be infinitely many solutions
   (g) Usually there will be no solution, but occasionally there will be one or more solutions
   (h) Usually there will be infinitely many solutions, but occasionally there will be one or no solutions

6. If one is solving three linear equations involving three unknowns, what happens?

   (a) There will always be exactly one solution
   (b) Usually there will be no solution, but occasionally there will be one or more solutions
   (c) Usually there will be infinitely many solutions, but occasionally there will be one or no solutions
   (d) Usually there will be one solution, but occasionally there will be no solutions or infinitely many solutions
   (e) There will always be a solution
   (f) There will always be infinitely many solutions
   (g) There will never be a solution
   (h) Anything can happen

7. What is the complete relationship between homogeneous linear systems of equations, and the zero solution (all unknowns equal to zero)?

   (a) The zero solution is always a solution to homogeneous linear systems, but could also be a solution to inhomogeneous linear systems

   (b) If a solution to a homogeneous linear system exists at all, then the zero solution will be a solution

   (c) The zero solution is always a solution to both homogeneous and inhomogeneous linear systems

   (d) The zero solution can be a solution to both homogeneous and inhomogeneous linear systems, but only if the equations are solvable

   (e) If a solution to a homogeneous linear system exists at all, it will be the zero solution

   (f) The zero solution is never a solution to inhomogeneous linear systems, and may or may not be a solution to homogeneous linear systems

   (g) The zero solution is always a solution to homogeneous linear systems, and never a solution to inhomogeneous linear systems

8. If one is solving three homogeneous equations involving two unknowns, what happens?

   (a) One usually has infinitely many solutions, but occasionally one just has only the zero solution

   (b) Usually one has no solutions, but occasionally one has one or infinitely many solutions

   (c) One has infinitely many solutions, including the zero solution

   (d) One can get different answers, depending on how you approach the problem

   (e) Usually the zero solution is the only solution, but occasionally one has more solutions

   (f) The zero solution is the only solution

   (g) There are no solutions

9. If one is solving two homogeneous equations involving three unknowns, what happens?

   (a) One has infinitely many solutions, including the zero solution

   (b) Usually one has no solutions, but occasionally one has one or infinitely many solutions

   (c) One can get different answers, depending on how you approach the problem

   (d) Usually the zero solution is the only solution, but occasionally one has more solutions

   (e) One usually has infinitely many solutions, but occasionally one just has only the zero solution

   (f) The zero solution is the only solution

   (g) There are no solutions

10. If one is solving three homogeneous equations involving three unknowns, what happens?

    (a) One has infinitely many solutions, including the zero solution
    (b) Usually one has no solutions, but occasionally one has one or infinitely many solutions
    (c) One can get different answers, depending on how you approach the problem
    (d) Usually the zero solution is the only solution, but occasionally one has more solutions
    (e) One usually has infinitely many solutions, but occasionally one just has only the zero solution
    (f) The zero solution is the only solution
    (g) There are no solutions

11. If a linear system has three equations in four unknowns, then

    (a) The rank of this system is twelve
    (b) The rank of this system can be three or four
    (c) The rank of this system is four
    (d) The rank of this system can be any number from zero to four
    (e) The rank of this system can be any number from zero to three
    (f) The rank of this system is three
    (g) The rank of the system is at least three

12. If a linear system has four equations in three unknowns, then

    (a) The rank of this system can be any number from zero to four
    (b) The rank of this system is three
    (c) The rank of this system is four
    (d) The rank of this system is twelve
    (e) The rank of this system can be any number from zero to three
    (f) The rank of this system can be three or four
    (g) The rank of the system is at least three

13. If a linear system has four unknowns and has rank three, then

    (a) There are infinitely many solutions, unless the system is inconsistent, in which case there are no solutions

    (b) There is either one solution or infinitely many solutions

    (c) There is either one solution or no solution

    (d) There are no solutions (system is inconsistent)

    (e) There are infinitely many solutions

    (f) There is exactly one solution

    (g) Anything can happen

14. If a linear system has four unknowns and has rank four, then

    (a) There are infinitely many solutions, unless the system is inconsistent, in which case there are no solutions

    (b) There is either one solution or infinitely many solutions

    (c) There is either one solution or no solution

    (d) There are no solutions (system is inconsistent)

    (e) There is exactly one solution

    (f) There are infinitely many solutions

    (g) Anything can happen

15. Let V be a vector space, and let W be a subset of V. What does it mean when we say that W is closed under addition?

    (a) If $x + y$ is in W, then x and y are in W

    (b) Whenever x and y are in W, then $x + y$ is in W

    (c) $W(x + y) = Wx + Wy$ for every two vectors x and y

    (d) Whenever x and y are in V, then $x + y$ is in W

    (e) Whenever x and y are in W, then $x + y$ is in V

    (f) Every vector in W is the sum of two vectors in W

    (g) Whenever x and y are in V, then $x + y$ is in V

16. Let V be a vector space, and let W be a subset of V. What does it mean when we say that W is closed under scalar multiplication?

    (a) If cx is in W and x is in W, then c is scalar
    (b) Whenever x is in W and c is a scalar, then cx is in V
    (c) Whenever x is in V and c is a scalar, then cx is in V
    (d) Whenever x is in V and c is a scalar, then cx is in W
    (e) $W(cx) = cWx$ for every vector x and scalar c
    (f) Whenever x is in W and c is scalar, then cx is in W
    (g) If cx is in W and c is scalar, then x is in W

17. The rank of a 3×5 matrix

    (a) Can be any number from zero to three
    (b) Can be any number from zero to two
    (c) Can be any number from two to five
    (d) Can be any number from zero to five
    (e) Must be two
    (f) Must be zero
    (g) Is three

18. The rank of a 5×3 matrix

    (a) Can be any number from zero to three
    (b) Can be any number from zero to two
    (c) Can be any number from two to five
    (d) Can be any number from zero to five
    (e) Must be two
    (f) Must be zero
    (g) Is three

19. Consider the system of linear equations

$$\begin{aligned} w + 3x + 2y + 2z &= 0 \\ w + 4x + y &= 0 \\ 3w + 5x + 10y + 14z &= 0 \\ 2w + 5x + 5y + 6z &= 0 \end{aligned}$$

    with solutions of the form (w, x, y, z) where w, x, y, and z are real. Which of the following statements is FALSE?

(a) The system is consistent

(b) The system has infinitely many solutions

(c) The sum of any two solutions is a solution

(d) $(-5, 1, 1, 0)$ is a solution

(e) Every solution is a scalar multiple of $(-5, 1, 1, 0)$

20. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 11 & 12 & 13 & 14 & 15 \\ 16 & 17 & 18 & 19 & 20 \\ 21 & 22 & 23 & 24 & 25 \end{bmatrix}$ is

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

21. Consider the following system of linear equations over the real numbers, where x, y, and z are variables and b is a real constant:

$$\begin{aligned} x + y + z &= 0 \\ x + 2y + 3z &= 0 \\ x + 3y + bz &= 0 \end{aligned}$$

Which of the following statements is true?

I. There exists a value of b for which the system has no solution

II. There exists a value of b for which the system has exactly one solution

III. There exists a value of b for which the system has more than one solution

(a) II only

(b) I and II only

(c) I and III only

(d) II and III only

(e) I, II, and III

22. Let $n \geq 1$. The $\mathbf{R}^n$ consists of

(a) n real numbers

(b) n-tuples of real numbers

(c) n-tuples of vectors

23. Which of the following statements is not an axiom for real vector spaces?

    (a) For all $x, y \in V$ we have $x + y = y + x$

    (b) For all $x, y, z \in V$, we have $(x + y) + z = x + (y + z)$

    (c) For all $x, y, z \in V$, we have $(xy)z = x(yz)$

24. Which formulation below can be completed correctly to give the definition of the concept of a real vector space?

    (a) A set V is called a real vector space if there exist two maps $+ : \mathbf{R} \times V \to V$, and $\cdot : \mathbf{R} \times V \to V$, so that the following eight axioms are satisfied...

    (b) A set of real vectors is called a real vector space, if the following eight axioms are satisfied...

    (c) A triple $(V, +, \cdot)$ in which V is a set and $+$ and $\cdot$ are respectively maps $V \times V \to V$ and $\mathbf{R} \times V \to V$ is called a real vector space if the following eight axioms are satisfied...

25. Which of the following statements is true? If V is a vector space over $\mathbf{R}$ then

    (a) $\{x + y \mid x \in V, y \in V\} = V$

    (b) $\{x + y \mid x \in V, y \in V\} = V \times V$

    (c) $\{\lambda u \mid \lambda \in \mathbf{R}, u \in V\} = \mathbf{R} \times V$

26. The rank of the real matrix $\begin{bmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{bmatrix}$

    (a) 1

    (b) 3

    (c) 5

27. For an $m \times n$ matrix A, we always have

    (a) rk $A \leq m$

    (b) $m \leq rkA \leq n$

    (c) $n \leq rkA$

28. A system of linear equations with coefficients in $\mathbf{R}$ is a system of equations of the following kind:

(a) $a_{11}x_1 + \cdots + a_{1n}x_1 = b_1$
$\vdots \qquad\qquad \vdots \quad \vdots \qquad$ with $a_{ij} \in \mathbf{R}, b_i \in \mathbf{R}$
$a_{n1}x_n + \cdots + a_{nn}x_n = b_n$

(b) $a_{11}x_{11} + \cdots + a_{1n}x_{1n} = b_1$
$\vdots \qquad\qquad \vdots \quad \vdots \qquad$ with $a_{ij} \in \mathbf{R}, b_i \in \mathbf{R}$
$a_{n1}x_{n1} + \cdots + a_{nn}x_{nn} = b_n$

(c) $a_{11}x_1 + \cdots + a_{1n}x_n = b_1$
$\vdots \qquad\qquad \vdots \quad \vdots \qquad$ with $a_{ij} \in \mathbf{R}, b_i \in \mathbf{R}$
$a_{n1}x_n + \cdots + a_{nn}x_n = b_n$

29. What is the solution to the following system of equations?

$$2x + y = 3$$
$$3x - y = 7$$

(a) $x = 4$ and $y = -5$
(b) $x = 4$ and $y = 5$
(c) $x = 2$ and $y = -1$
(d) $x = 2$ and $y = \frac{1}{2}$
(e) There are an infinite number of solutions to this system
(f) There are no solutions to this system

30. What is the solution to the following system of equations?

$$2x + y = 3$$
$$4x + 2y = 6$$

(a) $x = 0$ and $y = 0$
(b) $x = 2$ and $y = -1$
(c) $x = 0$ and $y = 1$
(d) $x = 0$ and $y = 3$
(e) There are an infinite number of solutions to this system
(f) There are no solutions to this system

31. What is the solution to the following system of equations?

$$-3x + 2y = 4$$
$$12x - 8y = 10$$

(a) $x = \frac{-4}{3}$ and $y = 0$
(b) $x = \frac{1}{2}$ and $y = \frac{-1}{2}$
(c) $x = 0$ and $y = 2$
(d) $x = \frac{1}{3}$ and $y = \frac{5}{2}$
(e) There are an infinite number of solutions to this system
(f) There are no solutions to this system

32. A system of linear equations could *not* have exactly _____ solutions

(a) 0
(b) 1
(c) 2
(d) infinite
(e) All of these are possible numbers of solutions to a system of linear equations

33. The system

$$x + y = 2$$
$$2x + 2y = 4$$

has an infinite number of solutions. Which of the following describes the set of solutions to this system?

(a) $x = 1$ and $y = 1$
(b) $x = 2 - t$ and $y = t$
(c) x and y can each be anything
(d) None of the above

34. Which of the following options describes the set of solutions to the system below?

$$x + y = 1$$
$$x - y = 0$$
$$2x + y = 3$$

(a) $x = 1 - t$ and $y = t$
(b) $x = 1$ and $y = 1$
(c) No solution exists
(d) None of the above

35. Which of the following options describes the set of solutions to the system below?

$$x + y = 2$$
$$2x - y = -2$$
$$x - 2y = -4$$

(a) $x = t$ and $y = 2 - t$

(b) $x = 0$ and $y = 2$

(c) No solution exists

(d) None of the above

36. A certain mini-golf course does not list their prices. I paid \$26.25 for 3 children and 4 adults. The group in front of me had paid \$25.50 for 6 children and 2 adults. Which system of equations would allow us to determine the prices for children and adults?

(a) $3x + 6y = 26.25$
$4x + 2y = 25.50$

(b) $3x + 4y = 26.25$
$6x + 2y = 25.50$

(c) $26.25x + 25.50y = 51.75$
$9x + 6y = 15$

(d) $\frac{26.25}{3}x + \frac{26.25}{4}y = 0$
$\frac{25.50}{6}x + \frac{25.50}{6}y = 0$

37. A system of 3 linear equations with 3 variables could not have exactly _____ solutions

(a) 0

(b) 1

(c) 2

(d) 3

(e) More than one of (a)-(d) are impossible

(f) All of (a)-(d) are possible numbers of solutions

38. A system of 5 linear equations and 7 variables could not have exactly _____ solutions.

(a) 0

(b) 1

(c) infinite

(d) More than one of these is impossible

(e) All of these are possible numbers of solutions

39. A system of 8 linear equations and 6 variables could not have exactly _____ solutions

(a) 0

(b) 1

(c) infinite

(d) More than one of these is impossible

(e) All of these are possible numbers of solutions

40. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your current inventory is 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs. Which matrix would best represent this information?

(a) $\begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 6 \\ 24 & 20 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 4 \\ 20 & 24 \end{bmatrix}$

(d) They all represent the information equally well

41. Which augmented matrix represents the following system of equations?

$$x + 2y = 3$$
$$4y + 5x = 6$$

(a) $\left[\begin{array}{cc|c} 0 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right]$

(b) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right]$

(c) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 5 & 4 & 6 \end{array}\right]$

(d) $\left[\begin{array}{cc|c} 0 & 2 & 3 \\ 5 & 4 & 6 \end{array}\right]$

42. Which matrix represents the following system of equations?

$$x = 6$$
$$y = 3$$

(a) $\begin{bmatrix} 1 & 6 \\ 1 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & 9 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 3 \end{bmatrix}$

43. What is the solution to the system of equations represented with this augmented matrix?

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

(a) $x = 2$, $y = 3$, $z = 4$

(b) $x = -1$, $y = 1$, $z = 1$

(c) There are an infinite number of solutions

(d) There is no solution

(e) We can't tell without having the system of equations

44. What is the solution to the system of equations represented with this augmented matrix?

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

(a) $x = 2$, $y = 3$, $z = 4$

(b) $x = -1$, $y = 1$, $z = 1$

(c) There are an infinite number of solutions

(d) There is no solution

(e) We can't tell without having the system of equations

45. What is the solution to the system of equations represented with this augmented matrix?

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(a) $x = 2$, $y = 3$, $z = 4$

(b) $x = -1$, $y = 1$, $z = 1$

(c) There are an infinite number of solutions

(d) There is no solution

(e) We can't tell without having the system of equations

46. Which of the following operations on an augmented matrix could change the solution set of a system?

(a) Interchanging two rows

(b) Multiplying one row by any constant

(c) Adding one row to an other

(d) Adding a multiple of one row to an other

(e) None of the above

(f) More than one of the above (specify which ones)

47. Which of the following matrices is NOT row equivalent to the one below? In other words, which matrix could you NOT get from the matrix below through elementary row operations?

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 3 & 4 \\ 1 & 2 & 0 & 4 \end{bmatrix}$$

(a) $\begin{bmatrix} 2 & 4 & 0 & 8 \\ 0 & 1 & 3 & 4 \\ 2 & 1 & 3 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 & 3 & 1 \\ 0 & 1 & 3 & 4 \\ 1 & 3 & 3 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 0 & 3 & 4 \\ 2 & 1 & 0 & 4 \end{bmatrix}$

(d) More than one of the above

(e) All are possible through elementary row operations

48. Which of the following matrices is row equivalent to the one below? In other words, which matrix could you get from the matrix below through elementary row operations?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) $\begin{bmatrix} 2 & 5 & 7 \\ 0 & 1 & 3 \\ 4 & 8 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 1 & 5 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 1 & 3 \\ -2 & 1 & 0 \\ 3 & 9 & 2 \end{bmatrix}$

(d) More than one of the above

(e) All are possible through elementary row operations

49. Which of the following matrices is NOT row equivalent to the one below? In other words, which matrix could you NOT get from the matrix below through elementary row operations?

$$\begin{bmatrix} 6 & 0 & 4 & 7 \\ 2 & 0 & 1 & 9 \\ 5 & 0 & 3 & 5 \end{bmatrix}$$

(a) $\begin{bmatrix} 12 & 0 & 8 & 14 \\ 2 & 0 & 1 & 9 \\ 1 & 0 & 1 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 12 & 0 & 8 & 14 \\ 0 & 0 & 1 & -20 \\ 2 & 1 & 3 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 0 & 4 & 7 \\ 2 & 0 & 1 & 9 \\ 7 & 0 & 4 & 14 \end{bmatrix}$

(d) All are possible through elementary row operations

50. What is the value of a so that the linear system represented by the following matrix would have infinitely many solutions?

$$\begin{bmatrix} 2 & 6 & 8 \\ 1 & a & 4 \end{bmatrix}$$

(a) $a = 0$

(b) $a = 2$

(c) $a = 3$

(d) $a = 4$

(e) This is not possible

(f) More than one of the above

51. Which of the following are solutions to the systems of equations?

$$\begin{aligned} 2x + y + 2z &= 0 \\ -x + 2y - 6z &= 0 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -4 \\ -2 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

52. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= 0 \\ x + 3y - 2z &= 0 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ 3 \\ 0 \end{bmatrix}$

(e) None of the above

(f) More than one of the above

53. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= 3 \\ x + 3y - 2z &= 4 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(d) None of the above

(e) More than one of the above

54. What is the solution to the following system of equations?

$$\begin{aligned} x + 2y + z &= -2 \\ x + 3y - 2z &= 1 \end{aligned}$$

(a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ -3 \\ 0 \end{bmatrix} + \begin{bmatrix} -7 \\ 3 \\ 1 \end{bmatrix} s$

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -8 \\ 3 \\ 0 \end{bmatrix} + \begin{bmatrix} 7 \\ -3 \\ -1 \end{bmatrix} s$

(d) None of the above

(e) More than one of the above

55. Consider a homogeneous linear system with n unknowns. Suppose the reduced row echelon form of its augmented matrix has $r \leq n$ nonzero rows. We can conclude that:

(a) $x_1 = 0$, $x_2 = 0, \ldots, x_n = 0$ is a solution to the system

(b) The system has $n - r$ free variables (parameters)

(c) The system has infinitely many solutions

(d) None of the above

(e) More than one of the above

56. Let $A = \begin{bmatrix} 5 & 4 & -8 & 1 \\ 1 & 3 & 4 & 8 \\ 0 & 2 & 1 & 3 \\ -1 & -2 & 4 & 1 \end{bmatrix}$. The reduced row echelon form of A is $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

What is the rank of A?

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

57. What size is this matrix? $\begin{bmatrix} 6 & 11 & -2 \\ 23 & 31 & 5 \end{bmatrix}$

(a) 2×3

(b) 3×2

(c) 6

58. A vector subspace does *not* have to satisfy which of the following properties?

(a) Associativity under vector addition

(b) Existence of an additive identity

(c) Commutativity under vector addition

(d) A vector subspace must satisfy all of the above properties

(e) A vector subspace need not satisfy any of the above properties

59. A vector subspace does *not* have to satisfy which of the following properties?

(a) Closure under vector addition

(b) Closure under scalar multiplication

(c) Closure under vector multiplication

(d) A vector subspace must satisfy all of the above properties

(e) A vector subspace need not satisfy any of the above properties

60. Consider the following system of equations:

$$\begin{aligned} x_1 + x_3 &= 3 \\ x_1 - x_2 - 3x_3 &= 1 \\ -x_1 + x_2 &= 4 \end{aligned}$$

The above system of linear equations is:

(a) inconsistent

(b) consistent with infinitely many solutions

(c) consistent with a unique solution

(d) None of the above

61. Consider the matrix $A = \begin{bmatrix} -1 & 1 & 2 & 1 & 2 \\ 0 & 1 & -3 & 1 & 4 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix}$ and let T be its corresponding linear map. Which of the following is true:

(a) The codomain of T is R^5

(b) T is one to one

(c) The range of T is the same as the codomain of T

(d) None of the above

62. Let R be the row reduced echelon form of the matrix $A = \begin{bmatrix} 1 & -2 & 0 & 2 \\ 2 & 2 & 2 & 1 \end{bmatrix}$. What is the value of R_{24}?

(a) 0

(b) $-\frac{1}{2}$

(c) $\frac{1}{3}$

(d) 2

(e) $-\frac{3}{8}$

63. The reduced row echelon form of the matrix

$$A = \begin{bmatrix} -2 & 2 & 3 & 1 & -3 & -2 \\ 0 & 2 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & -3 & -2 & 2 \end{bmatrix}$$

is given by

$$R = \begin{bmatrix} 1 & 0 & 0 & 10 & 2 & -7 \\ 0 & 1 & 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 1 & 7 & 0 & -5 \end{bmatrix}$$

What is the number of pivots of A?

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

(f) 6

64. Consider the system of equations

$$x_1 + x_3 = 3$$
$$x_1 - x_2 - x_3 = 1$$
$$-x_1 + x_2 = 4$$

This system has:

(a) No solution

(b) An infinite number of solutions

(c) A uniquely determined solution

(d) None of the above statements apply

65. What is the reduced row echelon form of the matrix $\begin{bmatrix} 0 & 1 & 2 & -1 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & -1 & 0 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

(e) None of the above

66. Determine the rank of the matrix $\begin{bmatrix} 0 & 0 & 1 & 2 & 1 \\ 2 & 4 & 0 & 2 & 0 \\ 3 & 6 & 2 & 7 & 2 \end{bmatrix}$

(a) 2

(b) 1

(c) 3

(d) 4

(e) 5

67. Consider the system of linear equations given by

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1$$
$$x_3 + x_4 + x_5 = 2$$
$$x_5 = 3$$

Determine how many parameters the solution set depends on

(a) 1 parameter

(b) 2 parameters

(c) 5 parameters

(d) 0 parameters

(e) 3 parameters

(f) There is no solution

68. Alice, John and Paul have purchased identical crayons and, while playing, got them all mixed up. Now the children need to divide 22 crayons between themselves. Luckily, Alice remembers that she had 3 more crayons than John did. Also, Paul remembers that he had as many crayons as the other two combined. How many crayons did Alice have?

(a) 2

(b) 5

(c) 7

(d) 2

(e) 12

(f) There is not enough data

69. Consider the augmented matrix $\left[\begin{array}{ccc|c} 1 & 3 & 3 & 5 \\ 3 & 3 & 2 & 1 \\ 4 & 0 & -2 & b \end{array}\right]$. For which value of b is the system consistent?

(a) -5

(b) 3

(c) -8

(d) 4

(e) 5

(f) It is consistent for all values of b

(g) It is inconsistent for all values of b

70. Consider the augmented matrix $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 4 & 7 \\ 4 & 5 & 9 \end{array}\right]$

Which of the following statements is true?

(a) There are more equations than unknowns

(b) There are fewer equations than unknowns

(c) The system is inconsistent

(d) There is an equivalent row echelon form having a pivot in every column of the system matrix

(e) The system has infinitely many solutions

(f) There are no free variables

(g) The system has a unique solution

(h) More than one of the above are true (Specify which ones)

71. Complete the following phrase to make a true statement: "A homogeneous linear system of 2011 linear equations in 1231 unknowns..."

(a) is always consistent

(b) always has a unique solution

(c) may be inconsistent

(d) which is consistent always has a unique solution

(e) which is consistent never has a unique solution

(f) is never consistent

72. The following system of equations

$$\begin{aligned} (4d-1)x + y + z &= 0 \\ -y + z &= 0 \\ (4d-1)z &= 0 \end{aligned}$$

has a non-trivial solution, if d equals

(a) 1/2

(b) 1/4

(c) 3/4

(d) 1

73. What is the rank of matrix $\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 3 & 2 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix}$

(a) 3

(b) 1

(c) 2

(d) 4