

# Chapter 1

1. (**True** | **False**) Every elementary row operation is reversible
2. (**True** | **False**) A  $5 \times 6$  matrix has six rows
3. (**True** | **False**) The solution set of a linear system involving variables  $x_1, \dots, x_n$  is a list of numbers  $(s_1, \dots, s_n)$  that makes each equation in the system a true statement when the values  $s_1, \dots, s_n$  are substituted for  $x_1, \dots, x_n$  respectively
4. (**True** | **False**) Two fundamental questions about a linear system involve existence and uniqueness
5. (**True** | **False**) Elementary row operations on an augmented matrix never change the solution set of the associated linear system
6. (**True** | **False**) Two matrices are row equivalent if they have the same number of rows
7. (**True** | **False**) An inconsistent system has more than one solution
8. (**True** | **False**) Two linear systems are equivalent if they have the same solution set
9. (**True** | **False**) In some cases, a matrix may be row reduced to more than one matrix in reduced row echelon form, using different sequences of row operations
10. (**True** | **False**) The row reduction algorithm applies only to augmented matrices for a linear system
11. (**True** | **False**) A basic variable in a linear system is a variable that corresponds to a pivot column in the coefficient matrix
12. (**True** | **False**) Finding a parametric description of the solution set of a linear system is the same as *solving* the system
13. (**True** | **False**) If one row in a row echelon form of an augmented matrix is  $[0 \ 0 \ 0 \ 5 \ 0]$ , then the associated linear system is inconsistent.
14. (**True** | **False**) The row echelon form of a matrix is unique
15. (**True** | **False**) The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process
16. (**True** | **False**) Whenever a system has free variables, the solution set contains many solutions
17. (**True** | **False**) A general solution of a system is an explicit description of all solutions of the system
18. (**True** | **False**) Another notation for the vector  $\begin{bmatrix} -4 \\ 3 \end{bmatrix}$  is  $[-4 \ 3]$
19. (**True** | **False**) The solution set of the linear system whose augmented matrix is  $[\mathbf{a}_1 \ \mathbf{a}_2 \ \mathbf{a}_3 \mid \mathbf{b}]$  is the same as the solution set of the equation  $x_1 \mathbf{a}_1 + x_2 \mathbf{a}_2 + x_3 \mathbf{a}_3 = \mathbf{b}$
20. (**True** | **False**) Any list of five real numbers is a vector in  $\mathbf{R}^5$

21. (**True** | **False**) The weights  $c_1, \dots, c_p$  in a linear combination  $c_1 \mathbf{v}_1 + \dots + c_p \mathbf{v}_p$  cannot all be zero
22. (**True** | **False**) A homogeneous equation is always consistent
23. (**True** | **False**) A linear transformation is a special type of function
24. (**True** | **False**) Every transformation  $F$  is linear if and only if  $F(c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2) = c_1 F(\mathbf{v}_1) + c_2 F(\mathbf{v}_2)$  for all  $\mathbf{v}_1$  and  $\mathbf{v}_2$  in the domain of  $T$  for all scalars  $c_1$  and  $c_2$
25. (**True** | **False**) If  $F : \mathbf{R}^n \rightarrow \mathbf{R}^m$  is a linear transformation and if  $\mathbf{c}$  is in  $\mathbf{R}^m$ , then a uniqueness question is “Is  $\mathbf{c}$  in in the range of  $T$ ?”
26. (**True** | **False**) A linear transformation  $F : \mathbf{R}^n \rightarrow \mathbf{R}^m$  is completely determined by its effect on the columns of the  $n \times n$  identity matrix
27. (**True** | **False**) When two linear transformations are performed one after another, the combined effect may not always be a linear transformation
28. (**True** | **False**) A mapping  $F : \mathbf{R}^n \rightarrow \mathbf{R}^m$  is onto  $\mathbf{R}^m$  if every vector  $\mathbf{x}$  in  $\mathbf{R}^n$  maps onto some vector in  $\mathbf{R}^m$
29. (**True** | **False**) A mapping  $F : \mathbf{R}^n \rightarrow \mathbf{R}^m$  is one-to-one if each vector in  $\mathbf{R}^n$  maps onto a unique vector in  $\mathbf{R}^m$
30. (**True** | **False**) Every matrix is row equivalent to a unique matrix in row echelon form.
31. (**True** | **False**) Any system of  $n$  linear equations in  $n$  variables has at most  $n$  solutions.
32. (**True** | **False**) If a system of linear equations has two different solutions, it must have infinitely many solutions.
33. (**True** | **False**) If a system of linear equations has no free variables, then it has a unique solution.
34. (**True** | **False**) If matrices  $A$  and  $B$  are row equivalent, they have the same reduced row echelon form.
35. (**True** | **False**) If an  $n \times n$  matrix  $A$  has  $n$  pivot positions, then the reduced row echelon form of  $A$  is the  $n \times n$  identity matrix.
36. (**True** | **False**) If  $3 \times 3$  matrices  $A$  and  $B$  each have three pivot positions, then  $A$  can be transformed into  $B$  by elementary row operations.
37. (**True** | **False**) If  $\mathbf{u}$ ,  $\mathbf{v}$ , and  $\mathbf{w}$ , are nonzero vectors in  $\mathbf{R}^2$ , then  $\mathbf{w}$  is a linear combination of  $\mathbf{u}$  and  $\mathbf{v}$ .
38. (**True** | **False**) If  $\mathbf{w}$  is a linear combination of  $\mathbf{u}$  and  $\mathbf{v}$  in  $\mathbf{R}^n$ , then  $\mathbf{u}$  is a linear combination of  $\mathbf{v}$  and  $\mathbf{w}$ .
39. (**True** | **False**) A linear transformation is a function.
40. (**True** | **False**) The range of a linear transformation is a vector space.
41. (**True** | **False**) Let  $A$  be an  $m \times n$  matrix. On a computer, row operations can change the apparent rank of a matrix.

42. (**True** | **False**) The rank of a matrix equals the number of nonzero rows.
43. (**True** | **False**) If  $B$  is obtained from a matrix  $A$  by several elementary row operations, then  $\text{rank } B = \text{rank } A$ .