Chapter 2

1. If M is the matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$, then M^{100} is

   (a) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (d) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

   (e) None of the above

2. If A is a 3×3 matrix such that $A \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $A \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, then the product $A \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$ is

   (a) $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

   (b) $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

   (d) $\begin{bmatrix} 9 \\ 10 \\ 11 \end{bmatrix}$

   (e) Not uniquely determined by the information given

3. $\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} =$

   (a) $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$

   (b) $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$

4. Let A and B be 2×3 matrices. Then

   (a) $A + B$ is a 2×3 matrix

   (b) $A + B$ is a 4×6 matrix

   (c) $A + B$ is a 4×9 matrix

5. For which of the following 3×3 matrices A do we have $AB = BA = B$ for all matrices B that are 3×3?

   (a) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

6. Which of the following matrix products is zero?

   (a) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -2 & -3 \end{bmatrix}$

   (b) $\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 3 & 3 \end{bmatrix}$

7. Which of the following properties does matrix multiplication lact

   (a) Associativity

   (b) Commutativity

   (c) Distributivity

8. For a matrix A that is $n \times n$, we ahve

   (a) If rank $A = n$, then A is invertible, but there exist invertible matrices with rank $A \neq n$

   (b) If A is invertible, then rank $A = n$, but there exist matrices A with rank $A = n$, which are not invertible

   (c) rank $A = n$ if and only if A is invertible

9. Which of the following transformations cannot be made elementary?

   (a) $\begin{bmatrix} 2 & 7 \\ 1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 7 \\ 3 & 8 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 1 \\ 2 & 7 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 2 \\ 7 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} -11 & 2 \\ 1 & 1 \end{bmatrix}$

10. Let A be an $m \times n$ matrix and B be a $n \times m$ matrix so that we have $\mathbf{R}^n \xrightarrow{A} \mathbf{R}^m \xrightarrow{B} \mathbf{R}^n$. Let $BA = E_n (= Id_{\mathbf{R}^n}$ as a linear map). Then

    (a) $m \geq n$, A injective, B surjective

    (b) $m \leq n$, A surjective, B injective

    (c) $m = n$, A and B invertible (bijective)

11. If one abbreviates a system of linear equations as $A\mathbf{x} = \mathbf{b}$, then

    (a) A is an $m \times n$ matrix and $b \in \mathbf{R}^n$

    (b) A is an $m \times n$ matrix and $b \in \mathbf{R}^m$

    (c) A is an $m \times n$ matrix and $b \in \mathbf{R}^n$ or $b \in \mathbf{R}^m$

12. A system of linear equations $A\mathbf{x} = \mathbf{b}$ is called solvable if

    (a) $A\mathbf{x} = \mathbf{b}$ for all $x \in \mathbf{R}^n$

    (b) $A\mathbf{x} = \mathbf{b}$ for precisely one $x \in \mathbf{R}^n$

    (c) $A\mathbf{x} = \mathbf{b}$ for at least one $x \in \mathbf{R}^n$

13. If $\mathbf{b}$ is one of the columns of A, then $A\mathbf{x} = \mathbf{b}$ is

    (a) solvable in all cases

    (b) unsolvable in all cases

    (c) sometimes solvable, sometimes unsolvable, depending on A and b

14. Let $A\mathbf{x} = \mathbf{b}$ be a system of equations with square matrix A (n equations in n unknowns). Then $A\mathbf{x} = \mathbf{b}$ is

   (a) uniquely solvable

   (b) solvable or unsolvable, depending on A, $\mathbf{b}$

   (c) solvable, but perhaps not uniquely, depending on A, $\mathbf{b}$

15. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If R is the reduced row echelon form of the augmented matrix for the system $A\mathbf{x} = \mathbf{b}$, what are the solutions to that system?

   (a) $x_1 = 1, x_2 = 1$, and $x_3 = 2$

   (b) $x_1 = 1, x_2 = 1, x_3 = 2$, and $x_4 = 0$

   (c) $x_1 = -t, x_2 = -t, x_3 = -2t$, and $x_4 = t$

   (d) There are no solutions to this system

16. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If R is the reduced row echelon form of the coefficent matrix for the system $A\mathbf{x} = \mathbf{0}$, what are the solutions to that system?

   (a) $x_1 = 1, x_2 = 1$, and $x_3 = 2$

   (b) $x_1 = 1, x_2 = 1, x_3 = 2$, and $x_4 = 0$

   (c) $x_1 = -t, x_2 = -t, x_3 = -2t$, and $x_4 = t$

   (d) There are no solutions to this system

17. For what value (or values) of m is the vector $(1, 2, m, 5)$ a linear combination of the vectors $(0, 1, 1, 1)$, $(0, 0, 0, 1)$, and $(1, 1, 2, 0)$?

   (a) For no value of m

   (b) -1 only

   (c) 1 only

   (d) 3 only

   (e) For infinitely many values of m

18. If $u = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$ and $v = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$, what is $2u - 3v$?

   (a) $\begin{bmatrix} -4 \\ 4 \\ 23 \end{bmatrix}$

(b) $\begin{bmatrix} 8 \\ 4 \\ -7 \end{bmatrix}$

(c) $\begin{bmatrix} 8 \\ 4 \\ 23 \end{bmatrix}$

(d) $\begin{bmatrix} 7 \\ 6 \\ 2 \end{bmatrix}$

19. Write $z = \begin{bmatrix} -5 \\ 3 \\ 16 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$ and $y = \begin{bmatrix} -3 \\ 2 \\ 6 \end{bmatrix}$

   (a) $z = -5x$
   (b) $z = -2x + y$
   (c) $z = x + 2y$
   (d) $z = 2x + y$
   (e) z cannot be written as a linear combination of x and y
   (f) None of the above

20. Write $z = \begin{bmatrix} 2 \\ -2 \\ 6 \end{bmatrix}$ as a linear combination of $x = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ and $y = \begin{bmatrix} 4 \\ -5 \\ 7 \end{bmatrix}$

   (a) $z = x + y$
   (b) $z = -x + y$
   (c) $z = 3x + 2y$
   (d) $z = -3x + y$
   (e) z cannot be written as a linear combination of x and y
   (f) None of the above

21. Suppose we have the vectors $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$. Which of the following is *not* a linear combination of these?

   (a) $\begin{bmatrix} 6 \\ 2 \\ 4 \end{bmatrix}$

   (b) $\begin{bmatrix} 8 \\ 0 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 8 \\ 1 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

(e) $\begin{bmatrix} 40 \\ 5 \\ 15 \end{bmatrix}$

(f) More than one of the above is not a linear combination of the given vectors

22. Suppose we have the vectors $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$. Which of the following is true?

(a) Every vector in $\mathbf{R}^3$ can be written as a linear combination of these vectors

(b) Some, but not all, vectors in $\mathbf{R}^3$ can be written as a linear combination of these vectors

(c) Every vector in $\mathbf{R}^2$ can be written as a linear combination of these vectors

(d) More than one of the above is true

(e) None of the above are true

23. Which of the following vectors can be written as a linear combination of the vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$?

(a) $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} .4 \\ 3.7 \end{bmatrix}$

(d) All of the above

24. Which of the following vectors can be written as a linear combination of the vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$?

(a) $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} .4 \\ 3.7 \\ -1.5 \end{bmatrix}$

(d) All of the above

25. Let z be any vector from $\mathbf{R}^3$. If we have a set V of unknown vectors from $\mathbf{R}^3$, how many vectors must be in V to guarantee that z can be written as a linear combination of the vectors in V?

(a) 2

(b) 3

(c) 4

(d) It is not possible to make such a guarantee

26. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. In this context, what interpretation can be given to the vector $15s_1$?

(a) $15s_1$ shows the number of people that can be served with 15 gallons of vanilla ice cream

(b) $15s_1$ shows the gallons of vanilla and chocolate ice cream sold by store 1 in 15 days

(c) $15s_1$ gives the total revenue from selling 15 gallons of ice cream at store 1

(d) $15s_1$ represents the number of days it will take to sell 15 gallons of ice cream at store 1

27. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. The stores are run by different managers, and they are not always able to be open the same number of days in a month. If store 1 is open for c_1 days in March, and store 2 is open for c_2 days in March, which of the following represents the total sales of each flavor of ice cream between the two stores?

(a) $c_1s_1 + c_2s_2$

(b) $\begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$

(c) $\begin{bmatrix} 5c_1 \\ 8c_1 \end{bmatrix} + \begin{bmatrix} 6c_2 \\ 10c_2 \end{bmatrix}$

(d) All of the above

(e) None of the above

28. Lucinda owns two ice cream parlors. The first ice cream shop sells 5 gallons of vanilla ice cream and 8 gallons of chocolate ice cream each day. The daily sales at the second store are 6 gallons of vanilla ice cream and 10 gallons of chocolate ice cream. The daily sales at stores one and two can be represented by the vectors $s_1 = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ and $s_2 = \begin{bmatrix} 6 \\ 10 \end{bmatrix}$, respectively. Lucinda is getting ready to close her ice cream parlors for the winter. She has a total of 39 gallons of vanilla ice cream in her warehouse, and 64 gallons of chocolate ice cream. She would like to distribute the ice cream to the two stores so that it is used up before the stores close for the winter. How much ice cream should she take to each store? The stores may stay open for different number of days, but no store may run out of ice cream before the end of the day on which it closes.

    (a) Lucinda should take 3 gallons of each kind of ice cream to store 1 and 4 gallons of each kind to store 2

    (b) Lucinda should take 3 gallons of vanilla to each store and 4 gallons of chocolate to each store

    (c) Lucinda should take 15 gallons of vanilla and 24 gallons of chocolate to store 1, and she should take 24 gallons of vanilla and 40 gallons of chocolate to store 2

    (d) Lucinda should take 15 gallons of vanilla and 32 gallons of chocolate to store 1, and she should take 18 gallons of vanilla and 40 gallons of chocolate to store 2

    (e) This cannot be done unless ice cream is thrown out or a store runs out of ice cream before the end of the day

29. Suppose a 4×4 matrix A has rank 4. How many solutions does the system $A\mathbf{x} = \mathbf{b}$ have?

    (a) 0

    (b) 1

    (c) Infinite

    (d) Not enough information given

30. Suppose a 4×4 matrix A has rank 3. How many solutions does the system $A\mathbf{x} = \mathbf{b}$ have?

    (a) 0

    (b) 1

    (c) Infinite

    (d) Not enough information given

31. Suppose a 4×4 matrix A has rank 3. If it is known that $(4, 5, 0, 1)$ is a solution to the system $A\mathbf{x} = \mathbf{b}$, then how many solutions does $Ax = b$ have?

    (a) 1

    (b) Infinite

    (c) Not enough information is given

32. Suppose a 5×5 matrix A has rank 3. If it is known that $(-1, 4, 2, 0, 3)$ is a solution to the system $A\mathbf{x} = \mathbf{b}$, then how many parameters does the solution set have?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4
(f) Not enough information given

33. Let $A = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$. What is $A + B$?

(a) 71

(b) $\begin{bmatrix} 6 & 9 \\ 7 & 11 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 11 \\ 23 & 31 \end{bmatrix}$

(d) $\begin{bmatrix} 26 & 62 \\ 112 & 268 \end{bmatrix}$

(e) $\begin{bmatrix} 4 & 6 & 2 & 5 \\ 20 & 24 & 3 & 7 \end{bmatrix}$

34. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$ what is A^T?

(a) $A^T = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$

(b) $A^T = \begin{bmatrix} 2 & 0 & -2 \\ 3 & -1 & 0 \\ 1 & 3 & 4 \end{bmatrix}$

(c) $A^T = \begin{bmatrix} -2 & 0 & 4 \\ 0 & -1 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

(d) $A^T = \begin{bmatrix} 1 & 3 & 4 \\ 3 & -1 & 0 \\ 2 & 0 & -2 \end{bmatrix}$

35. If $A = \begin{bmatrix} 4 & 6 \\ 20 & 7 \end{bmatrix}$ what is $5A$?

(a) $5A = \begin{bmatrix} 9 & 6 \\ 20 & 7 \end{bmatrix}$

(b) $5A = \begin{bmatrix} 9 & 11 \\ 25 & 12 \end{bmatrix}$

(c) $5A = \begin{bmatrix} 20 & 6 \\ 20 & 7 \end{bmatrix}$

(d) $5A = \begin{bmatrix} 20 & 30 \\ 100 & 35 \end{bmatrix}$

36. If A is a matrix and c a scalar such that $cA = 0$ (here 0 represents a matrix with all entries equal to zero), then

    (a) A is the identity matrix

    (b) $A = 0$

    (c) $c = 0$

    (d) Both $A = 0$ and $c = 0$

    (e) Either $A = 0$ and $c = 0$

    (f) We can't deduce anything

37. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, calculate the product AB

    (a) $AB = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

    (b) $AB = \begin{bmatrix} 10 & 7 \end{bmatrix}$

    (c) $AB = \begin{bmatrix} 8 & 4 \\ -3 & -2 \end{bmatrix}$

    (d) $AB = \begin{bmatrix} 7 \\ 10 \end{bmatrix}$

    (e) None of the above

    (f) This matrix multiplication is impossible

38. Calculate $\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$

    (a) $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

    (b) $\begin{bmatrix} 0 & -2 \\ 2 & 5 \end{bmatrix}$

    (c) $\begin{bmatrix} 0 & 0 \\ -6 & 2 \end{bmatrix}$

    (d) None of the above

    (e) This matrix multiplication is impossible

39. Calculate $\begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$

(a) $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & -2 \\ 2 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ 6 & -2 \end{bmatrix}$

(d) None of the above

(e) The matrix multiplication is impossible

40. If A and B are both 2×3 matrices, then which of the following is not defined?

(a) $A + B$

(b) $A^T B$

(c) BA

(d) AB^T

(e) More than one of the above

(f) All of these are defined

41. If A is a 2×3 matrix and B is a 3×6 matrix, what size is AB?

(a) 2×6

(b) 6×2

(c) 3×3

(d) 2×3

(e) 3×6

(f) This matrix multpication is impossible

42. In order to compute the matrix product AB, what must be true about the sizes of A and B?

(a) A and B must have the same number of rows

(b) A and B must have the same number of columns

(c) A must have as many rows as B has columns

(d) A must have as many columns as B has rows

43. If $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -1 & 3 \\ -2 & 0 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 & 2 \\ 1 & 2 & -1 \\ 3 & 1 & 0 \end{bmatrix}$ what is the AB_{32}? (You should be able to determine this without computing the entire matrix product)

(a) 1

(b) 3

(c) 4

(d) 8

44. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$ where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. If your October sales are 50% more than your May sales, which of the following would represent your October sales?

(a) $M + 50$

(b) $.5M$

(c) $1.5M$

(d) $M^{.5}$

45. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$ where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. Your June sales are given by the analogous matrix J, where $J = \begin{bmatrix} 6 & 8 \\ 22 & 32 \end{bmatrix}$. Which of the following matrix operations would make sense in this scenario? Be prepared to explain what the result tells you.

(a) $M + J$

(b) $M - J$

(c) $1.2J$

(d) MJ

(e) All of the above make sense

(f) More than one, but not all, of the above make sense

46. You have a business that sells tables and chairs. You have brown tables and white tables, and corresponding chairs. Your May sales are 4 brown tables, 6 white tables, 20 brown chairs, and 24 white chairs, which is represented by the matrix $M = \begin{bmatrix} 4 & 6 \\ 20 & 24 \end{bmatrix}$ where the first row is tables, the second row is chairs, the first column is brown items, and the second column is white items. All tables cost \$350 and all chairs cost \$125, which we represent with the cost vector $C = \begin{bmatrix} 350 \\ 125 \end{bmatrix}$. Which of the following matrix operations could be useful in this scenario? be prepared to explain what the result tells you.

(a) MC

(b) CM

(c) $C^T M$

(d) MC^T

47. Let A, B, C be 3 matrices such that the product ABC is defined. What is $(ABC)^T$?

(a) $(ABC)^T = A^T B^T C^T$

(b) $(ABC)^T = B^T C^T A^T$

(c) $(ABC)^T = C^T A^T B^T$

(d) $(ABC)^T = C^T B^T A^T$

48. Which of the following matrices does not have an inverse?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$

(e) More than one of the above do not have inverses

(f) All have inverses

49. When we put a matrix A into reduced row echelon form, we get the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$. This means that

(a) Matrix A has no inverse

(b) The matrix we have found is the inverse of matrix A

(c) Matrix A has an inverse, but this isn't it

(d) This tells us nothing about whether A has an inverse

50. Let $A = \begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$. What is A^{-1}?

(a) $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{2} & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{4} & 0 \end{bmatrix}$

51. We find that for a square coefficient matrix A, the homogenous matrix equation $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, has only the trivial solution $X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. This means that

(a) Matrix A has no inverse

(b) Matrix A has an inverse

(c) this tells us nothing about whether A has an inverse

52. We know that $(5A)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. What is matrix A?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$

(d) $\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$

(e) There is no matrix A which solves this equation

53. A and B are invertible matrices. If $AB = C$, then what is the inverse of C?

(a) $C^{-1} = A^{-1}B^{-1}$

(b) $C^{-1} = B^{-1}A^{-1}$

(c) $C^{-1} = AB^{-1}$

(d) More than one of the above is true

(e) Just because A and B have inverses, this doesn't mean that C has an inverse

54. Let A be a 2×2 matrix. The inverse of $3A$ is

(a) $\frac{1}{9}A^{-1}$

(b) $\frac{1}{3}A^{-1}$

(c) A^{-1}

(d) $3A^{-1}$

(e) Not enough information is given

55. If A is an invertible matrix, what else must be true?

(a) If $AB = C$ then $B = A^{-1}C$
(b) A^2 is invertible
(c) A^T is invertible
(d) $5A$ is invertible
(e) The reduced row echelon form of A is I
(f) All of the above must be true

56. The row-echelon reduced form of the matrix

$$A = \begin{bmatrix} -2 & 2 & 3 & 1 & -3 & -2 \\ 0 & 2 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & -3 & -2 & 2 \end{bmatrix}$$

is given by

$$R = \begin{bmatrix} 1 & 0 & 0 & 10 & 2 & -7 \\ 0 & 1 & 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 1 & 7 & 0 & -5 \end{bmatrix}$$

The number of free variables in the system of equations $A\mathbf{x} = 0$ is

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5
(f) 6

57. The row-echelon reduced form of the matrix

$$A = \begin{bmatrix} -2 & 2 & 3 & 1 & -3 & -2 \\ 0 & 2 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & -3 & -2 & 2 \end{bmatrix}$$

is given by

$$R = \begin{bmatrix} 1 & 0 & 0 & 10 & 2 & -7 \\ 0 & 1 & 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 1 & 7 & 0 & -5 \end{bmatrix}$$

Let F be the linear transformation $F\colon \mathbf{R}^6 \to \mathbf{R}^d$ given by $F(x) = Ax$. The number d is

(a) 1
(b) 2
(c) 3
(d) 4
(e) 5
(f) 6

58. Consider the matrix product AB where

$$A = \begin{bmatrix} -2 & 0 & -3 & 2 & -2 & -3 \\ 1 & -2 & -2 & -2 & 3 & -3 \\ 2 & 2 & 1 & 0 & -3 & 1 \\ 3 & 3 & -1 & 2 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 & -2 \\ -3 & -1 & -3 & -3 \\ -1 & 3 & -1 & -2 \\ 1 & 3 & 2 & -2 \\ -3 & -2 & -3 & 0 \\ -2 & -2 & -2 & -3 \end{bmatrix}$$

What is AB_{24}?

(a) 2
(b) -12
(c) 21
(d) -22
(e) $-13/12$

59. What is the first row of the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$?

(a) $\begin{bmatrix} 0 & -2 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} 0 & -3 & -6 \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 6 & -3 \end{bmatrix}$
(d) $\begin{bmatrix} 2 & 1 & 0 \end{bmatrix}$
(e) The inverse does not exist

60. Consider the system of linear equations:

$$2x_1 + 3x_2 + 4x_3 = 5$$
$$3x_1 + 2x_2 + x_3 = 0$$
$$x_1 + 2x_2 + 2x_3 = 2$$

Another way to express this system is:

(a) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} x_2 + \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} x_3 = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} x_1 + \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} x_2 + \begin{bmatrix} 1 & 2 & 2 \end{bmatrix} x_3 = \begin{bmatrix} 5 & 0 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 2 \end{bmatrix}$

61. Suppose that A is 3×4. Then the number of solutions to the system $A\mathbf{x} = 0$ is

(a) infinite

(b) one

(c) zero

(d) indeterminable without more information

62. If $A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$ and $AB = \begin{bmatrix} 1 & 0 & -2 & 5 \\ 4 & -1 & 0 & 3 \end{bmatrix}$, what is B_{24}?

(a) -4

(b) 8

(c) -2

(d) 3

(e) 11

63. Determine which of the following is not true for all $n \times n$ invertible matrices A, B, C

(a) $AB = CA \Rightarrow B = C$

(b) $(AB)^{-1} = B^{-1}A^{-1}$

(c) $(A^T)^{-1} = (A^{-1})^T$

(d) $A(B + C) = AB + AC$

(e) If $AB = AC$ then $B = C$

64. Let B be an $n \times n$ matrix. Suppose the equation $B\mathbf{x} = \mathbf{c}$ is inconsistent for some $\mathbf{c}$ in $\mathbf{R}^n$. Determine which of the following statements *must* be true.

(a) The equation $B\mathbf{x} = 0$ has more than one solution

(b) The columns of B span $\mathbf{R}^n$

(c) The linear transformation $\mathbf{x} \mapsto B\mathbf{x}$ is one-to-one

(d) The matrix B has n pivot positions

(e) The columns of B are linearly independent

65. Suppose that A and B are invertible matrices of the same size. Which of the following need not be invertible?

(a) $A+B$

(b) ABA^{-1}

(c) B^{-1}

(d) AB

(e) The identity matrix

(f) A^{-1}

66. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 1 \\ 2 & -2 & 1 \end{bmatrix}$. Find A^{-1} and give its first row.

(a) $(1, 0, -2)$

(b) $(-1, 2, -1)$

(c) $(1, 1, -1)$

(d) $(1, -2, 1)$

(e) $(2, -2, 1)$

(f) $(-1, 0, 2)$

67. Find a matrix A such that $\left(2A^T + \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\right)^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

and give its first row

(a) $(2, -1)$

(b) $(0, 0)$

(c) $(-1/2, 1/2)$

(d) $(0, 1/2)$

(e) $(1/2, 0)$

(f) $(1, 1/2)$

68. Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & t & 0 \\ 1 & 0 & -1 \end{bmatrix}$. Find the set of values of t for which the homogeneous system of linear equations $A\mathbf{x} = 0$ has a non-trivial solution

(a) $t = -3$

(b) $t \neq 2$

(c) $t \neq -3$

(d) $t \neq 1$ and $t \neq -3$

(e) $t = 1$ or $t = 3$

(f) $t = 2$

69. Which matrix product is defined?

(a) $\begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

70. Define $F(x) = Ax$ by the following matrix

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ -1 & 3 \end{bmatrix}$$

Which of the following vectors is not in the range of F?

(a) $\begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$

(b) $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} -1 \\ 0 \\ 5 \end{bmatrix}$

(d) $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

(e) $\begin{bmatrix}1\\4\\3\end{bmatrix}$

71. If $B = \begin{bmatrix}1 & 1 & -1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$, then the second row of B^{-1} is:

(a) $\begin{bmatrix}1 & 0 & -1\end{bmatrix}$
(b) $\begin{bmatrix}1 & 1 & 0\end{bmatrix}$
(c) $\begin{bmatrix}0 & 1 & -1\end{bmatrix}$
(d) $\begin{bmatrix}0 & -1 & 1\end{bmatrix}$
(e) $\begin{bmatrix}1 & -1 & 0\end{bmatrix}$
(f) None of the above

72. If three $n \times n$ matrices A, B, and C satisfy $AB - BA = C$,then ABA is always equal to:

(a) $A^2B - C$
(b) $A^2B - CA$
(c) $BA^2 + CA$
(d) A^2B
(e) $A^2B + AC$
(f) $A^2B + BC$

73. Compute $\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}^{2011}$

(a) $\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\2011 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$

(b) $\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & 2011\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2011 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

74. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & x \end{bmatrix}$. For which value(s) of x is A invertible?

(a) $x \neq -1$

(b) $x \neq 1$

(c) $x \neq 0$

(d) $x \neq -1$

(e) $x = 1$

(f) $x \neq \pm 1$

75. If the augmented matrix $[A|\mathbf{b}]$ of a system $A\mathbf{x} = \mathbf{b}$ is row equivalent to $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

Which of the following is true?

(a) The system is inconsistent

(b) $X = (5, -2 - s, 1)$ is the solution for any value of s

(c) $X = (5, -2, 1)$ is the unique solution of the system

(d) $X = (5s, -2s, s)$ is a solution for any value of s

(e) $X = (5t, -2 - s, s)$ is the solution for any value of s and t

(f) $X = (5, -3, 1)$ is the unique solution to the system

76. Suppose $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 4 \end{bmatrix}$

(a) A^{-1} does not exist

(b) The third row of A^{-1} is $\begin{bmatrix} -1 & -1 & 1 \end{bmatrix}$

(c) The second row of A^{-1} is $\begin{bmatrix} 1 & 2 & -1 \end{bmatrix}$

(d) The first row of A^{-1} is $\begin{bmatrix} 2 & 0 & -1 \end{bmatrix}$

(e) The second column of A^{-1} is $\begin{bmatrix}0 & 2 & -1\end{bmatrix}^T$

(f) All of B, C, D, and E are true

77. If $C = \begin{bmatrix}0 & 2 & 1\\ 1 & 0 & 1\end{bmatrix}$ and D is a $3 \times m$ matrix then the second row of the matrix CD is

(a) Not defined unless $m = 2$

(b) The same as the first row of D

(c) The same as the second row of D

(d) The sum of the first and the third row of D

(e) The sum of twice the second row of D and the third row of D

(f) Twice the first row of D

78. If C is a $n \times 4$ matrix and $D = \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$, then the second column of the matrix CD is

(a) The same as the second column of C

(b) The sum of the first and second columns of C

(c) The sum of the second and fourth columns of C

(d) The same as the first column of D

(e) The same as the third row of D

(f) The sum of the first and the third columns of C

79. The rank of a 3×3 matrix $C = AB$, found by multiplying a non-zero column matrix A of size 3×1 and a non-zero row matrix B of size 1×3, is

(a) 0

(b) 1

(c) 2

(d) 3

80. If A, B, C are square matrices of the same order, then $(ABC)^{-1}$ is equal to

(a) $C^{-1}A^{-1}B^{-1}$

(b) $C^{-1}B^{-1}A^{-1}$

(c) $A^{-1}B^{-1}C^{-1}$

(d) $A^{-1}C^{-1}B^{-1}$

81. Let A be a 3×4 matrix, and let B be a 4×3 matrix. Which of the four operations $A \times B$, $B \times A$, $A + B$, $A - B$, make sense?

(a) $A + B$ makes sense
(b) $A \times B$ makes sense
(c) $B \times A$ makes sense
(d) $A \times B$ and $B \times A$ make sense
(e) $A + B$ and $A - B$ make sense
(f) All of the operations make sense
(g) None of the operations make sense

82. Let A be a 3×4 matrix, and let B be a 4×5 matrix. Which of the four operations $A \times B$, $B \times A$, $A + B$, $A - B$, make sense?

(a) $A + B$ makes sense
(b) $A \times B$ makes sense
(c) $B \times A$ makes sense
(d) $A \times B$ and $B \times A$ make sense
(e) $A + B$ and $A - B$ make sense
(f) All of the operations make sense
(g) None of the operations make sense

83. Let A be a 3×4 matrix, and let B be a 3×4 matrix. Which of the four operations $A \times B$, $B \times A$, $A + B$, $A - B$, make sense?

(a) $A + B$ makes sense
(b) $A \times B$ makes sense
(c) $B \times A$ makes sense
(d) $A \times B$ and $B \times A$ make sense
(e) $A + B$ and $A - B$ make sense
(f) All of the operations make sense
(g) None of the operations make sense

84. Let A be a 3×3 matrix, and let B be a 3×3 matrix. Which of the four operations $A \times B$, $B \times A$, $A + B$, $A - B$, make sense?

   (a) $A + B$ makes sense
   (b) $A \times B$ makes sense
   (c) $B \times A$ makes sense
   (d) $A \times B$ and $B \times A$ make sense
   (e) $A + B$ and $A - B$ make sense
   (f) All of the operations make sense
   (g) None of the operations make sense

85. If one multiplies a row vector by a column vector, one gets

   (a) A number
   (b) A row vector
   (c) A column vector
   (d) An L-shaped vector
   (e) A matrix
   (f) Nothing; this operation cannot be defined in general
   (g) A number, if the two vectors have the same length, and nothing (undefined) otherwise

86. If one adds a row vector to a column vector, one gets

   (a) A number
   (b) A row vector
   (c) A column vector
   (d) An L-shaped vector
   (e) A matrix
   (f) Nothing; this operation cannot be defined in general
   (g) A number, if the two vectors have the same length, and nothing (undefined) otherwise

87. If one multiplies a matrix with a column vector, one gets

   (a) A number
   (b) A row vector
   (c) A column vector, if the number of rows of the matrix matches the number of rows of the vector
   (d) A column vector, if the number of columns of the matrix matches the number of rows of the vector
   (e) A column vector, if the number of rows of the matrix matches the number of columns of the vector
   (f) Nothing; this operation cannot be defined in general
   (g) A number, if the two vectors have the same length, and nothing (undefined) otherwise

88. If one multiplies a column vector with a row vector, one gets

(a) A number
(b) A matrix
(c) A row vector, in all cases
(d) A column vector, in all cases
(e) A row vector, if both vectors have the same length
(f) A column vector, if both vectors have the same length
(g) Nothing; this operation cannot be defined in general

89. If one multiplies a column vector with a matrix, one gets

(a) A number
(b) A matrix
(c) A row vector
(d) A column vector, if the number of rows of the matrix matches the number of rows of the vector
(e) A column vector, if the number of rows of the matrix matches the number of columns of the vector
(f) A column vector, if the number of columns of the matrix matches the number of rows of the vector
(g) Nothing; this operation cannot be defined in general

90. Let A be a matrix. Under what conditions will $A \times A$ make sense?

(a) A must have at least as many rows as columns
(b) A must be a column vector
(c) A must have at least as many columns as rows
(d) A must be in reduced row-echelon form
(e) $A \times A$ makes sense for any matrix A
(f) A must be a square matrix
(g) A must be a row vector

91. Let $F : \mathbf{R}^3 \to \mathbf{R}^5$ be a linear transformation. Then

(a) F is invertible if and only if the rank is five
(b) F is one-to-one if and only if the rank is two; F is never onto
(c) F is onto if and only if the rank is three; F is never one-to-one
(d) F is one-to-one if and only if the rank is five; F is never onto
(e) F is one-to-one if and only if the rank is three; F is never onto
(f) F is onto if and only if the rank is two; F is never one-to-one
(g) F is onto if and only if the rank is five; F is never one-to-one

92. Let $F : \mathbf{R}^5 \to \mathbf{R}^3$ be a linear transformation. Then

(a) F is one-to-one if and only if the rank is five; F is never onto

(b) F is onto if and only if the rank is five; F is never one-to-one

(c) F is onto if and only if the rank is two; F is never one-to-one

(d) F is invertible if and only if the rank is five

(e) F is one-to-one if and only if the rank is three; F is never onto

(f) F is onto if and only if the rank is three; F is never one-to-one

(g) F is one-to-one if and only if the rank is two; F is never onto

93. Let A be an invertible 5×5 matrix. Which of the following statements is *false*?

(a) The linear transformation associated to A must be both one-to-one and onto

(b) The row-reduced echelon form of A must contain no free variables

(c) Every row of A must contain a leading 1

(d) For every vector $\mathbf{b}$ in $\mathbf{R}^5$, there must be exactly one solution to the equation $A\mathbf{x} = \mathbf{b}$

(e) The rank of A must equal 5

(f) The reduced row echelon form of A must be the identity matrix

(g) There must exist a 5×5 matrix B, such that $AB = BA = I$