Chapter 2

Directions: For questions 1 - 23, mark each statement True or False. Justify each answer.

1. (**True** | **False**) The vector $\mathbf{u}$ results when a vector $\mathbf{u} - \mathbf{v}$ is added to the vector $\mathbf{v}$
2. (**True** | **False**) An example of a linear combination of vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is the vector $\frac{1}{2}\mathbf{v}_1$
3. (**True** | **False**) The equation $A\mathbf{x} = \mathbf{b}$ is referred to as a *vector equation*
4. (**True** | **False**) A vector $\mathbf{b}$ is a linear combination of the columns of a matrix A if and only if the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution
5. (**True** | **False**) The equation $A\mathbf{x} = \mathbf{b}$ is consistent if the augmented matrix $\left[\begin{array}{c|c} A & \mathbf{b} \end{array}\right]$ has a pivot position in every row
6. (**True** | **False**) The first entry in the product $A\mathbf{x}$ is a sum of products
7. (**True** | **False**) If the columns of an $m \times n$ matrix A span $\mathbf{R}^m$, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for each $\mathbf{b}$ in $\mathbf{R}^m$
8. (**True** | **False**) If A is an $m \times n$ and if the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b}$ in $\mathbf{R}^m$, then A cannot have a pivot position in every row
9. (**True** | **False**) Every matrix with equation $A\mathbf{x} = \mathbf{b}$ corresponds to a vector equation with the same solution set
10. (**True** | **False**) Any linear combination of vectors can always be written in the form $A\mathbf{x}$ for a suitable matrix A and vector $\mathbf{x}$
11. (**True** | **False**) The solution set of a linear system whose augmented matrix is $\left[\begin{array}{ccc|c} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{b} \end{array}\right]$ is the same as the solution set of $A\mathbf{x} = \mathbf{b}$, if $A = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 \end{bmatrix}$
12. (**True** | **False**) If the augmented matrix $\left[\begin{array}{c|c} A & \mathbf{b} \end{array}\right]$ has a pivot position in every row, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent
13. (**True** | **False**) If A is an $m \times n$ matrix whose columns do not span $\mathbf{R}^m$, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b}$ in $\mathbf{R}^m$
14. (**True** | **False**) The equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if the equation has at least one free variable
15. (**True** | **False**) If $\mathbf{x}$ is a nontrivial solution of $A\mathbf{x} = \mathbf{0}$, then every entry in $\mathbf{x}$ is nonzero
16. (**True** | **False**) The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is a solution
17. (**True** | **False**) The solution set of $A\mathbf{x} = \mathbf{b}$ is obtained by translating the solution set of $A\mathbf{x} = \mathbf{0}$
18. (**True** | **False**) If A is a 3×5 matrix and F is a transformation defined by $F(\mathbf{x}) = A\mathbf{x}$, then the domain of F is $\mathbf{R}^3$

19. (**True** | **False**) If A is an $m \times n$ matrix, then the range of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is $\mathbf{R}^m$

20. (**True** | **False**) The codomain of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the columns of A

21. (**True** | **False**) A linear transformation preserves the operations of vector addition and scalar multiplication

22. (**True** | **False**) If A is a 3×2 matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot be one-to-one

23. (**True** | **False**) If A is a 3×2 matrix, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot map $\mathbf{R}^2$ onto $\mathbf{R}^3$

Directions: For questions 24 - 32, mark each statement True or False. Justify each answer. If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case.

24. (**True** | **False**) If an augmented matrix $\left[\begin{array}{c|c} A & \mathbf{b} \end{array}\right]$ is transformed into $\left[\begin{array}{c|c} A & \mathbf{b} \end{array}\right]$ by elementary row operations, then the equations $A\mathbf{x} = \mathbf{b}$ and $C\mathbf{x} = \mathbf{d}$ have exactly the same solution sets.

25. (**True** | **False**) If a system $A\mathbf{x} = \mathbf{b}$ has more than one solution, then so does the system $A\mathbf{x} = \mathbf{0}$.

26. (**True** | **False**) If an augmented matrix $\left[\begin{array}{c|c} A & \mathbf{b} \end{array}\right]$ can be transformed by elementary row operations into reduced row echelon form, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent.

27. (**True** | **False**) The equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if there are no free variables.

28. (**True** | **False**) If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbf{R}^m$, then A has m pivot columns.

29. (**True** | **False**) If an $m \times n$ matrix A has a pivot position in every row, then the equation $A\mathbf{x} = \mathbf{b}$ has a unique solution for each $\mathbf{b}$ in $\mathbf{R}^m$.

30. (**True** | **False**) If A s an $m \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ has at least two different solutions, and if the equation $A\mathbf{x} = \mathbf{c}$ is consistent, then the equation $A\mathbf{x} = \mathbf{c}$ has many solutions.

31. (**True** | **False**) If A is a 6×5 matrix, the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ cannot map $\mathbf{R}^5$ onto $\mathbf{R}^6$.

32. (**True** | **False**) If A is an $m \times n$ matrix with m pivot columns, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is a one-to-one mapping.

Directions: Questions 33 - 42 concern arbitrary matrices A, B, and C for which for which the indicated sums and products are defined. Mark each statement True or False.

33. (**True** | **False**) If A and B are 2×2 with columns $\mathbf{a}_1, \mathbf{a}_2$, and $\mathbf{b}_1, \mathbf{b}_2$ respectively, then $AB = \begin{bmatrix} \mathbf{a}_1 \mathbf{b}_1 & \mathbf{a}_2 \mathbf{b}_2 \end{bmatrix}$

34. (**True** | **False**) Each column of AB is a linear combination of the columns of B using weights from the corresponding column of A

35. (**True** | **False**) $AB + BC = A(B + C)$

36. (**True** | **False**) $A^T + B^T = (A + B)^T$

37. (**True** | **False**) The transpose of a product of matrices equals the product of their transposes in the same order

38. (**True** | **False**) If A and B are 3×3 and $B = \begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 \end{bmatrix}$, then $AB = \begin{bmatrix} A\,\mathbf{b}_1 + A\,\mathbf{b}_2 + A\,\mathbf{b}_3 \end{bmatrix}$

39. (**True** | **False**) The second row of AB is the second row of A multiplied on the right by B

40. (**True** | **False**) $(AB)C = (AC)B$

41. (**True** | **False**) $(AB)^T = A^T B^T$

42. (**True** | **False**) The transpose of a sum of matrices equals the sum of their transposes

Directions: For questions 43 - 56, mark each statement True or False. Justify each answer.

43. (**True** | **False**) In order for a matrix B to be the inverse of A, both equations $AB = I$ and $BA = I$ must be true.

44. (**True** | **False**) If A and B are $n \times n$ and invertible, then $A^{-1}B^{-1}$ is the inverse of AB

45. (**True** | **False**) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ab - cd \neq 0$, then A is invertible

46. (**True** | **False**) If A is an invertible $n \times n$ matrix, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent for *each* in $\mathbf{R}^n$

47. (**True** | **False**) Each elementary matrix is invertible.

48. (**True** | **False**) A product of invertible $n \times n$ matrices is invertible, and the inverse of the product is the product of their inverses in the same order

49. (**True** | **False**) If A is invertible, then the inverse of A^{-1} is A itself

50. (**True** | **False**) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $ad = bc$, then A is not invertible

51. (**True** | **False**) If A can be row reduced to the identity matrix, then A must be invertible

52. (**True** | **False**) If A is invertible, then elementary row operations that reduce A to the identity I_n also reduce A^{-1} to I_n

53. (**True** | **False**) If $A = \begin{bmatrix} A_1 & A_2 \end{bmatrix}$ and $B = \begin{bmatrix} B_1 & B_2 \end{bmatrix}$, with A_1 and A_2 the same sizes as B_1 and B_2, respectively, then $A + B = \begin{bmatrix} A_1 + B_1 & A_2 + B_2 \end{bmatrix}$

54. (**True** | **False**) If $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and $B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$, then AB exists

55. (**True** | **False**) The definition of the matrix–vector product $A\mathbf{x}$ is a special case of matrix multiplication

56. (**True** | **False**) If A_1, A_2, B_1, and B_2 are $n \times n$ matrices, $A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$, and $B = \begin{bmatrix} B_1 & B_2 \end{bmatrix}$ then the product BA is defined, but AB is not

Directions: Assume that the matrices mentioned in the statements for questions 61 - 72 have appropriate sizes. Mark each statement True or False.

57. (**True** | **False**) If A and B are $m \times n$, then both AB^T and A^TB are defined.

58. (**True** | **False**) If $AB = C$ and C has 2 columns, then A has 2 columns.

59. (**True** | **False**) Left-multiplying a matrix B by a diagonal matrix A, with nonzero entries on the diagonal, scales the rows of B.

60. (**True** | **False**) If $BC = BD$, then $C = D$

61. (**True** | **False**) If $AC = 0$, then either $A = 0$ or $C = 0$

62. (**True** | **False**) If A and B are $n \times n$, then $(A + B)(A - B) = A^2 - B^2$.

63. (**True** | **False**) An elementary $n \times n$ matrix has either n or $n + 1$ nonzero entries.

64. (**True** | **False**) The transpose of an elementary matrix is an elementary matrix.

65. (**True** | **False**) An elementary matrix must be square.

66. (**True** | **False**) Every square matrix is a product of elementary matrices.

67. (**True** | **False**) If A is a 3×3 matrix with three pivot positions, there exist elementary matrices $E_1, \ldots, E_p$ such that $E_p \cdots E_1 A = I$.

68. (**True** | **False**) If $AB = I$, then A is invertible.

69. (**True** | **False**) If A and B are square and invertible, then AB is invertible, and $(AB)^{-1} = A^{-1}B^{-1}$.

70. (**True** | **False**) If $AB = BA$ and if A is invertible, then $A^{-1}B = BA^{-1}$.

71. (**True** | **False**) If A is invertible and if r $\neq 0$, then $(rA)^{-1} = rA^{-1}$.

72. (**True** | **False**) If A is a 3×3 matrix and the equation $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ has a unique solution, then A is invertible.

For questions 73- 81, mark each statement True or False. Justify each answer.

73. (**True** | **False**) Let matrix R be the reduced row echelon form of matrix A. The solutions to $R\mathbf{x} = \mathbf{0}$ are the same as the solutions to $A\mathbf{x} = \mathbf{0}$

74. (**True** | **False**) Let matrix R be the reduced row echelon form of matrix A. The solutions to $R\mathbf{x} = \mathbf{b}$ are the same as the solutions to $A\mathbf{x} = \mathbf{b}$

75. (**True** | **False**) Suppose y and z are both solutions to $A\mathbf{x} = \mathbf{b}$. All linear combinations of y and z also solve $A\mathbf{x} = \mathbf{b}$

76. (**True** | **False**) Suppose y and z are both solutions to $A\mathbf{x} = \mathbf{0}$. All linear combinations of y and z also solve $A\mathbf{x} = \mathbf{0}$

77. (**True** | **False**) If $AX = BX$ for all matrices X where the products are defined, then A and B have to be the same matrix

78. (**True** | **False**) If $A\mathbf{x} = B\mathbf{x}$ for all vectors $\mathbf{x}$ where the products are defined, then A and B have to be the same matrix

79. (**True** | **False**) If A and B are square matrices with the same dimensions, then $(A + B) \times (A + B) = A^2 + 2AB + B^2$

80. (**True** | **False**) If A, B, and C are square matrices and we know that $AB = AC$, this means that matrix B is equal to matrix C

81. (**True** | **False**) Suppose that A, B, and C are square matrices, and $CA = B$, and A is invertible. This means that $C = A^{-1}B$.