Chapter 3

1. The dimension of the subspace spanned by the real vectors

$$\begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\2\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}2\\0\\0\\3\end{bmatrix}, \begin{bmatrix}1\\-2\\0\\8\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \text{ is}$$

   (a) 2
   (b) 3
   (c) 4
   (d) 5
   (e) 6

2. Suppose B is a basis for a real vector space V of dimension greater than 1. Which of the following statements could be true?

   (a) The zero vector of V is an element of B
   (b) B has a proper subset that spans V
   (c) B is a proper subset of a linearly independent subset of V
   (d) There is a basis for V that is disjoint from B
   (e) One of the vectors in B is a linear combination of the other vectors in B

3. If V_1 and V_2 are 6-dimensional subspaces of a 10-dimensional vector space V, what is the smallest possible dimension that $V_1 \cap V_2$ can have?

   (a) 0
   (b) 1
   (c) 2
   (d) 4
   (e) 6

4. If V and W are 2-dimensional subspaces of $\mathbf{R}^4$, what are the possible dimensions of the subspace $V \cap W$?

   (a) 1 only
   (b) 2 only
   (c) 0 and 1 only
   (d) 0, 1, and 2 only
   (e) 0, 1, 2, 3, and 4

5. Let V and W be 4-dimensional subspaces of a 4-dimensional vector space X. Which of the following CANNOT be the dimension of the subspace $V \cap W$?

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

6. Which of the following statements is true?

   (a) If U is a subspace of V, then $V \cap U'$ is also a subspace of V
   (b) There exists a subspace U of V for which $V \cap U'$ is also a subspace, but $V \cap U'$ is not a subspace for all subspaces U
   (c) If U is a subspace of V, then $V \cap U'$ is never a subspace of V

7. Which of the following subsets $U \subset \mathbf{R}^n$ is a subspace?

   (a) $U = \{x \in \mathbf{R}^n \mid x_1 = \cdots = x_n\}$
   (b) $U = \{x \in \mathbf{R}^n \mid x_1^2 = x_2^2\}$
   (c) $U = \{x \in \mathbf{R}^n \mid x_1 = 1\}$

8. How many subspaces does $\mathbf{R}^2$ have?

   (a) two: 0 and $\mathbf{R}^2$
   (b) four: 0, $\mathbf{R} \times 0$, $0 \times \mathbf{R}$ (the "axes"), and $\mathbf{R}^2$ itself
   (c) infinitely many

9. For which of the following objects does the description "linearly dependent" or "linearly independent" make sense?

   (a) An list $(\mathbf{v}_1, \ldots, \mathbf{v}_n)$ of elements of a vector space
   (b) An list $(V_1, \ldots, V_n)$ of vector spaces
   (c) A linear combination $\lambda_1 \mathbf{v}_1 + \cdots + \lambda_n \mathbf{v}_n$, where the λ_i are real numbers and the $\mathbf{v}_i$ are elements of a vector space

10. Let $\mathbf{v}_1, \ldots, \mathbf{v}_n \in V$. What does $\mathrm{Span}(\mathbf{v}_1, \ldots, \mathbf{v}_n) = V$ mean?

    (a) Each linear combination $\lambda_1 \mathbf{v}_1 + \cdots + \lambda_n \mathbf{v}_n$ is an element of V
    (b) Each element of V is a linear combination of $\lambda_1 \mathbf{v}_1 + \cdots + \lambda_n \mathbf{v}_n$
    (c) The dimension of V is n

11. For linearly independent triples $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ of vectors in a vector space V,

(a) $(\mathbf{v}_1, \mathbf{v}_2)$ is always linearly dependent

(b) $(\mathbf{v}_1, \mathbf{v}_2)$ may or may not be linearly dependent, depending on the choice of $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$

(c) $(\mathbf{v}_1, \mathbf{v}_2)$ is always linearly independent

12. Which of the following statements implies the linear independence of the n-tuple $(\mathbf{v}_1, \ldots, \mathbf{v}_n)$ of elements of V?

    (a) $\lambda_1 \mathbf{v}_1 + \cdots + \lambda_n \mathbf{v}_n = 0$ only if $\lambda_1 = \lambda_2 = \cdots = \lambda_n = 0$

    (b) If $\lambda_1 = \cdots = \lambda_n = 0$, then $\lambda_1 \mathbf{v}_1 + \cdots + \lambda_n \mathbf{v}_n = 0$

    (c) $\lambda_1 \mathbf{v}_1 + \cdots + \lambda_n \mathbf{v}_n = 0$ for all $(\lambda_1, \ldots, \lambda_n) \in F^n$

13. The vector space $V = \{\mathbf{0}\}$ consisting of only the zero vector

    (a) has the basis $\{\mathbf{0}\}$

    (b) has the basis $\varnothing$

    (c) has no basis

14. If one were to define $U_1 - U_2 := \{x - y \mid x \in U_1, y \in U_2\}$ for subspaces U_1, U_2 of V, the one would have

    (a) $U_1 - U_1 = \{0\}$

    (b) $(U_1 - U_2) + U_2 = U_1$

    (c) $U_1 - U_2 = U_1 + U_2$

15. A map $f : V \to W$ between vector spaces V and W over $\mathbf{R}$ is linear, if

    (a) $f(\lambda x + \mu y) = \lambda f(x) + \mu f(y)$ for all $x, y \in V, \lambda, \mu \in \mathbf{R}$

    (b) f satisfies the eight axioms for a vector space

    (c) $f : V \to W$ is bijective

16. The definition of the kernel of a linear map between vector spaces $f\colon V \to W$ is

    (a) $\{w \in W \mid f(0) = w\}$

    (b) $\{f(v) \mid v = 0\}$

    (c) $\{v \in V \mid f(v) = 0\}$

17. Which of the following statements are correct? If $f : V \to W$ is a linear map, we have

    (a) $f(0) = 0$

    (b) $f(-x) = -x$ for all $x \in V$

    (c) $f(\lambda v) = f(\lambda) + f(v)$ for all $\lambda \in \mathbf{R}$, $v \in V$

18. Let V and W be two vector spaces with bases $(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3)$ and (w_1, w_2, w_3) and let $f : V \to W$ be the linear map with $f(\mathbf{v}_i) = w_i$. Then the "associated" matrix is

(a) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

19. A linear map $f : V \to W$ is injective if and only if

    (a) f is surjective

    (b) $\dim \text{Ker } f = 0$

    (c) $\text{rk } f = 0$

20. Let $f : V \to W$ be a surjective linear map and $\dim V = 5$, $\dim W = 3$. Then

    (a) $\dim \text{Ker } f \geq 3$

    (b) $\dim \text{Ker } f$ is 0, 1, or 2 and each of these cases can arise

    (c) $\dim \text{Ker } f = 2$

21. Which set of vectors is linearly independent?

    (a) (2,3),(8,12)

    (b) (1,2,3),(4,5,6),(7,8,9)

    (c) $(-3,1,0)$,(4,5,2),(1,6,2)

    (d) None of these sets are linearly independent

    (e) Exactly two of these sets are linearly independent

    (f) All of these sets are linearly independent

22. Suppose you wish to determine whether a set of vectors is linearly independent. You form a matrix with those vectors as the columns, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. What do you decide?

    (a) These vectors are linearly independent

    (b) These vectors are not linearly independent

23. Suppose you wish to determine whether a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent. You form the matrix $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \end{bmatrix}$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write $\mathbf{v}_4$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$. Which is a correct linear combination?

(a) $\mathbf{v}_4 = \mathbf{v}_1 + \mathbf{v}_2$

(b) $\mathbf{v}_4 = -\mathbf{v}_1 - 2\mathbf{v}_3$

(c) $\mathbf{v}_4$ cannot be written as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$

(d) We cannot determine the linear combination from this information

24. Suppose you wish to determine whether a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent. You form the matrix $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \end{bmatrix}$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write $\mathbf{v}_3$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_4$. Which is a correct linear combination?

(a) $\mathbf{v}_3 = \frac{1}{2}\mathbf{v}_1 - \frac{1}{2}\mathbf{v}_4$

(b) $\mathbf{v}_3 = \frac{1}{2}\mathbf{v}_1 + \frac{1}{2}\mathbf{v}_2$

(c) $\mathbf{v}_3 = 2\mathbf{v}_1 + 3\mathbf{v}_2$

(d) $\mathbf{v}_3 = -2\mathbf{v}_1 - 3\mathbf{v}_2$

(e) $\mathbf{v}_3$ cannot be written as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_4$

(f) We cannot determine the linear combination from this information

25. Suppose you wish to determine whether a set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent. You form the matrix $A = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 \end{bmatrix}$, and you calculate its reduced row echelon form, $R = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. You now decide to write $\mathbf{v}_2$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_3$, and $\mathbf{v}_4$. Which is a correct linear combination?

(a) $\mathbf{v}_2 = 3\mathbf{v}_3 + \mathbf{v}_4$

(b) $\mathbf{v}_2 = -3\mathbf{v}_3 - \mathbf{v}_4$

(c) $\mathbf{v}_2 = \mathbf{v}_4 - 3\mathbf{v}_3$

(d) $\mathbf{v}_2 = -\mathbf{v}_1 + \mathbf{v}_4$

(e) $\mathbf{v}_2$ cannot be written as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_3$, and $\mathbf{v}_4$

(f) We cannot determine the linear combination from this information

26. Are the vectors $\begin{bmatrix} 1 \\ 4 \\ 5 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} -14 \\ 13 \\ 7 \\ -19 \end{bmatrix}$ linearly independent?

(a) Yes, they are linearly independent

(b) No, they are not linearly independent

27. To determine whether a set of n vectors from $\mathbf{R}^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

    (a) A row of all zeros

    (b) A row that has all zeros except in the last position

    (c) A column of all zeros

    (d) An identity matrix

28. To determine whether a set of fewer than n vectors from $\mathbf{R}^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

    (a) An identity submatrix with zeros below it

    (b) A row that has all zeros except in the last position

    (c) A column that is not an identity matrix column

    (d) A column of all zeros

29. If the columns of A are not linearly independent, how many solutions are there to the system $A\mathbf{x} = \mathbf{0}$?

    (a) 0

    (b) 1

    (c) Infinite

    (d) Not enough information given

30. Which statement is equivalent to saying that $\mathbf{v}_1, \mathbf{v}_2, and\, \mathbf{v}_3$ are linearly independent vectors?

    (a) The only solution to $c_1\,\mathbf{v}_1 + c_2\,\mathbf{v}_2 + c_3\,\mathbf{v}_3 = 0$ is $c_1 = c_2 = c_3 = 0$

    (b) $\mathbf{v}_3$ cannot be written as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$

    (c) No vector is a multiple of any other

    (d) Exactly two of (a), (b), and (c) are true.

    (e) All three statements are true

31. Suppose a 4×4 matrix A has rank 4. Are the columns of A linearly independent?

    (a) Yes, they are linearly independent

    (b) No, they are not linearly independent

    (c) We do not have enough information to decide

32. Which property of vector spaces is not true for the following set?

$$\left(\begin{bmatrix}-2\\-2\end{bmatrix}, \begin{bmatrix}-1\\-1\end{bmatrix}, \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}2\\2\end{bmatrix} \right)$$

(a) Closure under vector addition
(b) Existence of an additive identity
(c) Existence of an additive inverse for each vector
(d) None of the above

33. The set of all 2×2 matrices with determinant equal to zero is not a vector subspace. Why?

(a) 2×2 matrices are not vectors
(b) With matrices, AB need not equal BA
(c) $\begin{bmatrix}1 & 1\\1 & 1\end{bmatrix} + \begin{bmatrix}1 & 2\\1 & 1\end{bmatrix} = \begin{bmatrix}2 & 3\\2 & 2\end{bmatrix}$ and $\begin{bmatrix}2 & 3\\2 & 2\end{bmatrix}$ is not in the set
(d) $\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} + \begin{bmatrix}0 & 1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ and $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ is not in the set
(e) None of the above

34. Which of the following sets of vectors is a basis for $\mathbf{R}^3$

(a) $\{(1,0,0),(0,1,0),(0,0,1)\}$
(b) $\{(1,0,1),(1,1,0),(1,1,1)\}$
(c) $\{(2,0,0),(0,5,0),(0,0,8)\}$
(d) All are bases for $\mathbf{R}^3$

35. Let $\mathbf{v}_1 = \begin{bmatrix}1\\1\\2\end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix}3\\0\\-1\end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix}6\\0\\-2\end{bmatrix}$. Which of the following sets has the same span as the set of all three vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

(a) $\{\mathbf{v}_1, \mathbf{v}_2\}$
(b) $\{\mathbf{v}_2, \mathbf{v}_3\}$
(c) $\{\mathbf{v}_1, \mathbf{v}_3\}$
(d) None of the above
(e) More than one of the above

36. Let $\mathbf{v}_1 = \begin{bmatrix}1\\1\\2\end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix}3\\0\\-1\end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix}6\\0\\-2\end{bmatrix}$. Which of the following vectors is *not* in the subspace of $\mathbf{R}^3$ spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

(a) (1,0,0)

(b) (4,1,1)

(c) (3,3,6)

(d) All of these are in the subspace of $\mathbf{R}^3$ spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

37. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, w = \begin{bmatrix} k \\ 2 \\ -3 \end{bmatrix}$. For how many values k will the vector w be in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

(a) No values of k - vector w will never be in this subspace

(b) Exactly one value of k will work

(c) Any value of k will work

38. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, w = \begin{bmatrix} k \\ 8 \\ 11 \end{bmatrix}$. For how many values k will the vector w be in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

(a) No values of k - vector w will never be in this subspace

(b) Exactly one value of k will work

(c) Any value of k will work

39. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, w = \begin{bmatrix} 1 \\ k \\ k \end{bmatrix}$. For how many values k will the vector w be in the subspace spanned by $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

(a) No values of k - vector w will never be in this subspace

(b) Exactly one value of k will work

(c) Any value of k will work

40. How many linearly independent columns are there in the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) 2

(b) 1

(c) 0

41. The *column space* of a matrix A is the set of vectors that can be created by taking all linear combinations of the columns of A. Is the vector $b = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$ in the column space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) Yes, since we can find a vector x so that $Ax = b$

(b) Yes, since $-2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} - 1 \begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$

(c) No, because there is no vector x so that $Ax = b$

(d) No, because we can't find c_1 and c_2 such that $c_1 \begin{bmatrix} 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} -4 \\ 12 \end{bmatrix}$

(e) More than one of the above

(f) None of the above

42. The column space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is

(a) The set of all linear combinations of the columns of A

(b) A line in $\mathbf{R}^2$

(c) The set of all multiples of the vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

(d) All of the above

(e) None of the above

43. How many solutions x are there to $Ax = 0$ where $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

(a) 0 solutions

(b) 1 solution

(c) 2 solutions

(d) Infinite number of solutions

44. The *null space* of a matrix A is the set of all vectors x that are solutions of $Ax = 0$. Which of the following vectors is in the null space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) $x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(b) $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(c) $x = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$

(d) All of the above

(e) None of the above

45. The *row space* of a matrix A is the set of vectors that can be created by taking all linear combinations of the rows of A. Which of the following vectors is in the row space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$?

(a) $x = \begin{bmatrix} -2 & 4 \end{bmatrix}$

(b) $x = \begin{bmatrix} 4 & 8 \end{bmatrix}$

(c) $x = \begin{bmatrix} 0 & 0 \end{bmatrix}$

(d) $x = \begin{bmatrix} 8 & 4 \end{bmatrix}$

(e) More than one of the above

(f) None of the above

46. The row space of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ consists of

(a) All linear combinations of the columns of A^T

(b) All multiples of the vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(c) All linear combinations of the rows of A

(d) All of the above

(e) None of the above

47. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. Which of the following vectors are in the nullspace of A?

(a) $\begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \\ -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} 3 \\ -1 \\ 3 \\ 2 \end{bmatrix}$

48. Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. How many vectors are in the nullspace of A?

(a) Only one

(b) Probably more than one, but it's hard to say how many

(c) An infinite number

49. To determine whether a set S of vectors is linearly independent, you form a matrix which has those vectors as columns, and you calculate its reduced row echelon form. Suppose the resulting form is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. How many linearly independent vectors are in S?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

50. To determine whether a set S of vectors is linearly independent, you form a matrix which has those vectors as columns, and you calculate its reduced row echelon form. Suppose the resulting form is $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. Which of the following subsets of S are linearly independent?

(a) The first, second, and third vectors
(b) The first, second, and fourth vectors
(c) The first, third, and fourth vectors
(d) The second, third, and fourth vectors
(e) All of the above
(f) More than one, but not all, of the above

51. Write $d = (3, -5, 10)$ as a linear combination of the vectors $a = (-1, 0, 3), b = (0, 1, 5)$, and $c = (4, -2, 0)$

(a) $d = -3a - 5b + c$
(b) $d = 5a - b + 2c$
(c) $d = (10/3)a + (5/2)c$
(d) d cannot be written as a linear combination of a, b, and c

52. Which of the following sets of vectors spans $\mathbf{R}^3$?

i.) $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$

ii.) $\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

iii.) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix}$

iv.) $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

(a) i, ii, iii, and iv

(b) ii, iii, and iv only
(c) ii and iii only
(d) i, ii and iii only
(e) iii and iv only
(f) ii only

53. Which of the following sets of vectors forms a basis for $\mathbf{R}^3$?

i.) $\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix}$

ii.) $\begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

iii.) $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix}$

iv.) $\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

(a) i, ii, iii, and iv
(b) ii, iii, and iv only
(c) ii and iii only
(d) i, ii, and iii only
(e) iii and iv only
(f) ii only

54. Which of the following describes a basis for a subspace V?

(a) A basis is a linearly independent spanning set for V
(b) A basis is a minimal spanning set for V
(c) A basis is a largest possible set of linearly independent vectors in V
(d) All of the above
(e) Some of the above
(f) None of the above

55. Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 3 & -1 & 0 & 2 \end{bmatrix}$. The reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. What is the dimension of the column space of A?

(a) 1
(b) 2
(c) 3

(d) 4

56. Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 3 & -1 & 0 & 2 \end{bmatrix}$. The reduced row echelon form of A is $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Which columns would form a basis for the columns space of A?

(a) All four
(b) The first three
(c) Any three
(d) Any two

57. Let $B = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. Which of the following describes the column space of B?

(a) The columns space of B is all of $\mathbf{R}^3$
(b) The column space of B is a proper subset of $\mathbf{R}^3$
(c) The column space of B is $\mathbf{R}^4$
(d) The columns space of B is a proper subset of $\mathbf{R}^4$
(e) None of the above

58. Let $B = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 2 & -1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. What is the dimension of the column space of B?

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4
(f) Infinite

59. Let $A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 3 & 1 \\ 2 & -1 & 1 & 1 \end{bmatrix}$. What is the dimension of the nullspace of A?

(a) 0
(b) 1
(c) 2
(d) 3

(e) 4
(f) Infinite

60. Let A be an $n \times n$ matrix. If A is an invertible matrix, what else must be true?

(a) The columns of A form a basis of $\mathbf{R}^n$
(b) The rank of A is n
(c) The dimension of the column space of A is n
(d) The dimension of the null space of A is 0
(e) All of the above must be true
(f) More than one, but not all, of the above have to be true

61. Which of the following sets is linearly independent?

(a) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -12 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

62. Let $v = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$, $w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and $0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ Which of the following are *linearly dependent*? Select all that apply

(a) $\{0\}$
(b) $\{v\}$
(c) $\{w, 0\}$
(d) $\{v, 0\}$
(e) $\{v, w\}$
(f) $\{v, w, 0\}$

63. The dimension of the subspace of a $2/times2$ matrix spanned by $\begin{bmatrix} 1 & -5 \\ -4 & 2 \end{bmatrix}$, $\begin{bmatrix} 1 & 1 \\ -1 & 5 \end{bmatrix}$ and $\begin{bmatrix} 2 & -4 \\ -5 & 7 \end{bmatrix}$ is

(a) 1
(b) 2
(c) 3
(d) 4

64. Let u, v, w be three non-zero vectors which are linearly independent, then

(a) u is linear combination of v and w
(b) v is linear combination of u and w
(c) w is linear combination of u and v
(d) None of these

65. Which of the following statements are true?

I. A set $\{u, v, w\}$ of vectors is linearly independent if only if for scalars a,b,$c \in \mathbf{R}$, $au + bv + cw = 0$ implies $a = b = c = 0$

II. A set $\{u, v, w\}$ of vectors is linearly independent if and only if for scalars a,b,$c \subset \mathbf{R}$, $au + bv + cw = 0$ when $a = b = c = 0$

III. A set $\{u, v, w\}$ of vectors is linearly independent if and only if u is not a linear combination of v and w

IV. $\{(1, -1), (1, 1)\}$ spans R^2

V. $\{(1, 0, 1), (1, 1, 1), (2, 1, 2)\}$ spans R^3

(a) I & IV
(b) II & IV
(c) I & II
(d) III & V
(e) III & II
(f) I & V

66. Let A be a 11×6 matrix such that $Ax = 0$ has only the trivial solution $x = 0$. What is the rank of A? Do the columns of A span $\mathbf{R}^{11}$?

(a) 0, Yes
(b) 6, Yes
(c) 6, No
(d) 8, Yes
(e) 8, No
(f) 2, Yes

67. For which value of α does the vector (2,3,α) belong to the subspace of $\mathbf{R}^3$ spanned by (1,0,3) and (3,2,1)?

(a) 0
(b) 6
(c) -6
(d) 1
(e) -1
(f) 1/2

68. A vector space V has dimension 15 and W is a subspace of V in which $\{\mathbf{v}_1, ..., \mathbf{v}_8\}$ is a spanning set. Which of the following statements are always true?

I. dim $W < 8$

II. dim $W \leq 15$

III. dim $W \leq 7$

IV. Any linearly independent set in W has no more than 8 vectors in it

(a) (I) and (II)
(b) (I) and (III)
(c) (II) and (IV)
(d) (II) and (III)
(e) (I), (III) and (IV)
(f) (III) and (IV)

69. For what value of α is the set of vectors $\{(1,1,1),(1,2,0),(2,3,\alpha)\}$ linearly dependent?

(a) -1
(b) 2
(c) 0
(d) 1
(e) $-1/2$
(f) -2

70. If A is a 7×12 matrix, what is the smallest possible dimension of the kernel of A?

(a) 0
(b) 3
(c) 5
(d) 7
(e) 11
(f) 12

71. Find the value of t for which $(4, 6, 3, t)$ belongs to Span $\{(1, 3, -4, 1), (2, 8, -5, -1), (-1, -5, 0, 2)\}$

(a) 0
(b) 4
(c) 7
(d) 11
(e) 13
(f) 15

72. For what value of c is the set of vectors $\{(1, c, 1), (0, 1, 1), (1, 0, 2)\}$ linearly dependent?

(a) -2
(b) -1
(c) $-1/2$
(d) 0
(e) 1
(f) 2

73. In a vector space V , suppose $\{u, v\}$ is linearly independent and w is such that $\{u, v, w\}$ is linearly dependent. Which of the following is *ALWAYS* true?

(a) $u \in \text{Span}\{v, w\}$
(b) $v \in \text{Span}\{u, w\}$
(c) $w \in \text{Span}\{u, v\}$
(d) $\{u, u + v, w\}$ is linearly independent
(e) $\{u, w\}$ is linearly dependent
(f) $\{v, w\}$ is linearly dependent

74. If $\{u, v, w\}$ is a set of vectors in a vector space V, and a, b, and c are scalars, which of the following statements are true?

I. If none of the vectors u, v or w is a multiple of any other vector in $\{u, v, w\}$, then $\{u, v, w\}$ is linearly independent

II. If $au + bv + cw = 0$ can occur only when $a = b = c = 0$, then $\{u, v, w\}$ is linearly independent

III. If $a = b = c = 0$ implies $au + bv + cw = 0$, then $\{u, v, w\}$ is linearly independent

IV. If $au + bv + cw = 0$ implies $a = b = c = 0$,then $\{u, v, w\}$ is linearly independent

(a) III & IV
(b) II & IV
(c) I & IV

(d) II & III

(e) I & III

(f) I & II

75. A set of linear equations is represented by the matrix equation $Ax = b$. The necessary condition for the existence of a solution for this system is

(a) A must be invertible

(b) b must be linearly dependent on the columns of A

(c) b must be linearly independent of the columns of A

(d) None of these

76. If A is a 3×5 matrix, then the rank of A is

(a) A 3×5 matrix

(b) A 5×3 matrix

(c) A number (possibly non-zero)

(d) A subspace of $\mathbf{R}^3$

(e) A subspace of $\mathbf{R}^5$

(f) Zero

(g) Undefined

77. If A is a 3×5 matrix, then the transpose of A is

(a) A 3×5 matrix

(b) A 5×3 matrix

(c) A number (possibly non-zero)

(d) A subspace of $\mathbf{R}^3$

(e) A subspace of $\mathbf{R}^5$

(f) Zero

(g) Undefined

78. If A is a 3×5 matrix, then the inverse of A is

(a) A 3×5 matrix

(b) A 5×3 matrix

(c) A number (possibly non-zero)

(d) A subspace of $\mathbf{R}^3$

(e) A subspace of $\mathbf{R}^5$

(f) Zero

(g) Undefined

79. If A is a 3×5 matrix, then the image of A is
    (a) A 3×5 matrix
    (b) A 5×3 matrix
    (c) A number (possibly non-zero)
    (d) A subspace of $\mathbf{R}^3$
    (e) A subspace of $\mathbf{R}^5$
    (f) Zero
    (g) Undefined

80. If A is a 3×5 matrix, then the kernel of A is
    (a) A 3×5 matrix
    (b) A 5×3 matrix
    (c) A number (possibly non-zero)
    (d) A subspace of $\mathbf{R}^3$
    (e) A subspace of $\mathbf{R}^5$
    (f) Zero
    (g) Undefined

81. If A is a 3×5 matrix, then the row reduced-echelon form of A is
    (a) A 3×5 matrix
    (b) A 5×3 matrix
    (c) A number (possibly non-zero)
    (d) A subspace of $\mathbf{R}^3$
    (e) A subspace of $\mathbf{R}^5$
    (f) Zero
    (g) Undefined

82. If one applies row reduction to a matrix, then
    (a) The image, rank, and kernel may change, but the nullity does not change
    (b) The image, kernel, and nullity may change, but the rank does not change
    (c) The image may change, but the kernel, rank, and nullity do not change
    (d) The kernel may change, but the image, rank, and nullity do not change
    (e) The image, kernel, rank, and nullity may all change

(f) The image, kernel, rank, and nullity all do not change

(g) The image and kernel may change, but the rank and nullity do not change

83. If one replaces a matrix with its transpose, then

(a) The image, rank, and kernel may change, but the nullity does not change

(b) The image, kernel, and nullity may change, but the rank does not change

(c) The image may change, but the kernel, rank, and nullity do not change

(d) The kernel may change, but the image, rank, and nullity do not change

(e) The image, kernel, rank, and nullity may all change

(f) The image, kernel, rank, and nullity all do not change

(g) The image and kernel may change, but the rank and nullity do not change

84. Let $T : \mathbf{R}^4 \to \mathbf{R}^4$ be the transformation $T(x_1, x_2, x_3, x_4) = (0, x_1, x_2, x_3)$. The null space (or kernel) $N(T)$ of T consists of all vectors of the form

(a) $(x_1, x_2, x_3, 0)$, where x_1, x_2, x_3, are real numbers

(b) $(0, x_1, x_2, x_3)$, where x_1, x_2, x_3, are real numbers

(c) $(0, 0, 0, 1)$

(d) $(x_1, 0, 0, 0)$, where x_1 is a real number

(e) $(0, 0, 0, x_4)$, where x_4 is a real number

(f) $(x_4, 0, 0, 0)$, where x_4 is a real number

(g) $(1, 0, 0, 0)$

85. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation $T(x_1, x_2) = (x_1, 0)$. The null space (or kernel) $N(T)$ of T is

(a) $(0, x_2)$

(b) $(0, x_2) : x_2$ is real

(c) 1

(d) $(1, 0)$

(e) $(x_1, 0)$

(f) $(x_1, 0) : x_1$ is real

(g) $(0, 1)$

86. Let $T : \mathbf{R}^4 \to \mathbf{R}^4$ be the transformation $T(x_1, x_2, x_3, x_4) = (x_2, x_3, 0, 0)$. The null space (or kernel) $N(T)$ of T consists of all vectors of the form

(a) $(1, 0, 0, 0)$ and $(0, 1, 0, 0)$

(b) $(0, x_2, x_3, 0)$, where x_2 and x_3 are real numbers

(c) $(x_1, 0, 0, x_4)$, where x_1 and x_4 are real numbers

(d) $(x_2, x_3, 0, 0)$, where x_2 and x_3 are real numbers

(e) $(1, 0, 0, 0)$ and $(0, 0, 0, 1)$

(f) $(0, 0, x_1, x_4)$, where x_1 and x_4 are real numbers

(g) $(0, 0, x_3, x_4)$, where x_3 and x_4 are real numbers

87. Let $T : \mathbf{R}^3 \to \mathbf{R}^4$ be the transformation $T(x_1, x_2, x_3) = (x_1, 0, x_2, x_2)$. The range $R(T)$ of T has many bases; one of them is the set of vectors

(a) $(1, 0, 0)$ and $(0, 1, 0)$

(b) $(1, 0, 0, 0)$, $(0, 0, 1, 0)$, and $(0, 0, 0, 1)$

(c) $(1, 0, 0, 0)$ and $(0, 0, 1, 1)$

(d) $(x_1, 0, 0)$ and $(0, x_2, 0)$

(e) $(1, 0, 0, 0)$, $(0, 1, 0, 0)$, $(0, 0, 1, 0)$ and $(0, 0, 0, 1)$

(f) $(x_1, 0, 0)$, $(0, x_2, 0)$ and $(0, 0, x_3)$

(g) $(x_1, 0, 0, 0)$, $(0, 0, x_2, 0)$, and $(0, 0, 0, x_2)$

88. Let $T : \mathbf{R}^3 \to \mathbf{R}^4$ be the transformation $T(x_1, x_2, x_3) = (x_1, 0, x_2, x_2)$. The null space (or kernel) $N(T)$ of T has many bases; one of them is the set of vectors

(a) $(1, 0, 0)$ and $(0, 1, 0)$

(b) $(x_1, x_2, 0)$

(c) $(0, 0, 1)$

(d) $(1, 0, 0, 0)$ and $(0, 0, 1, 1)$

(e) $(0, 0, x_3)$

(f) $(0, 1, 0, 0)$

89. Let $T : \mathbf{R}^4 \to \mathbf{R}^4$ be the transformation $T(x_1, x_2, x_3, x_4) = (0, x_1, x_2, x_3)$. The image $Im(T)$ of T consists of all vectors of the form

(a) $(0, 1, 0, 0)$, $(0, 0, 1, 0)$, $(0, 0, 0, 1)$

(b) (x_1, x_2, x_3, x_4), where x_1, x_2, x_3, x_4 are real numbers

(c) $(x_4, 0, 0, 0)$, where x_4 is a real number

(d) $(x_1, x_2, x_3, 0)$ where x_1, x_2, and x_3 are real numbers

(e) $(0, x_1, x_2, x_3)$ where x_1, x_2, and x_3 are real numbers

(f) $(0, 0, 0, x_4)$, where x_4 is a real number

90. A transformation $T : \mathbf{R}^n \to \mathbf{R}^m$ is linear if and only if

(a) One has $T(x + y) = Tx + Ty$ and $T(cx) = cT(x)$ for all vectors x, y, and scalars c
(b) The image of T is a line
(c) The graph of T takes the form $y = mx + c$
(d) No condition required (all transformations are linear)
(e) T is one-to-one and onto
(f) There exists a matrix A such that $Tx = Ax$ for all $x \epsilon \mathbf{R}^n$
(g) One has $T(x + y) = Tx + Ty$ for all vectors x, y

91. If a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^5$ is one-to-one, then

(a) The rank is three and the nullity is zero
(b) The rank is three and the nullity is two
(c) The rank is two and the nullity is three
(d) The rank is five and the nullity is two
(e) The rank and nullity can be any pair of non-negative numbers that add up to three
(f) The rank and nullity can be any pair of non-negative numbers that add up to five
(g) The situation is impossible

92. The linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^5$ is onto, then

(a) The rank is three and the nullity is zero
(b) The rank is five and the nullity is two
(c) The rank is three and the nullity is two
(d) The rank is two and the nullity is three
(e) The rank and nullity can be any pair of non-negative numbers that add up to three
(f) The rank and nullity can be any pair of non-negative numbers that add up to five
(g) The situation is impossible

93. If the linear transformation $T : \mathbf{R}^5 \to \mathbf{R}^3$ is onto, then

(a) The rank is two and the nullity is three
(b) The rank is five and the nullity is two
(c) The rank is three and the nullity is zero
(d) The rank and nullity can be any pair of non-negative numbers that add up to three
(e) The rank and nullity can be any pair of non-negative numbers that add up to five
(f) The rank is three and the nullity is two
(g) The situation is impossible

94. The linear transformation $T : \mathbf{R}^5 \to \mathbf{R}^3$ is one-to-one, then

(a) The rank and nullity can be any pair of non-negative numbers that add up to five
(b) The rank is five and the nullity is two
(c) The rank is three and the nullity is two
(d) The rank is two and the nullity is three
(e) The rank is three and the nullity is zero
(f) The rank and nullity can be any pair of non-negative numbers that add up to three
(g) The situation is impossible

95. Let $T : \mathbf{R}^5 \to \mathbf{R}^3$ be a linear transformation. Then

(a) T is invertible if and only if the nullity is zero
(b) T is onto if and only if the nullity is three; T is never one-to-one
(c) T is onto if and only if the nullity is zero; T is never one-to-one
(d) T is one-to-one if and only if the nullity is zero; T is never onto
(e) T is one-to-one if and only if the nullity is three; T is never onto
(f) T is one-to-one if and only if the nullity is two; T is never onto
(g) T is onto if and only if the nullity is two; T is never one-to-one

96. Let $T : \mathbf{R}^3 \to \mathbf{R}^5$ be a linear transformation. Then

(a) T is invertible if and only if the nullity is zero
(b) T is onto if and only if the nullity is three; T is never one-to-one
(c) T is onto if and only if the nullity is zero; T is never one-to-one
(d) T is one-to-one if and only if the nullity is zero; T is never onto
(e) T is one-to-one if and only if the nullity is three; T is never onto
(f) T is one-to-one if and only if the nullity is two; T is never onto
(g) T is onto if and only if the nullity is two; T is never one-to-one

97. Let V be a vector space, and let S be a subset of V. What does it mean when we say that S is linearly independent?

(a) The number of elements S is less than or equal to the dimension of V
(b) The only way to write 0 as a linear combination of elements of S is the zero combination (where one takes zero multiples of each element of S)
(c) S is closed under both addition and scalar multiplication
(d) All the elements of S are distinct from each other
(e) Every element in V is a linear combination of elements in S

(f) S is a basis

(g) S has nullity zero

98. Let V be a vector space, and let S be a subset of V. What does it mean when we say that S is linearly dependent?

   (a) S is closed under both addition and scalar multiplication

   (b) At least two of the elements of S are the same

   (c) Every element of S is a linear combination of other elements S

   (d) There is a way to write 0 as a linear combination of elements S other than the zero combination

   (e) The number of elements of S is greater than the dimension of V

   (f) S depends on a linear transformation

   (g) The span of S has smaller dimension than the dimension of V

99. Let V be a vector space, and let S be a subset of V. What does it mean when we say that S spans V?

   (a) The rank of S is the same as the dimension of V

   (b) The elements of S are all distinct from each other

   (c) Every vector in V has exactly one representation as a linear combination of vectors in S

   (d) S has at least as many elements as the dimension V

   (e) S is a basis for V

   (f) Every vector in V can be expressed as a linear combination of vectors in S

100. Let V be a five-dimensional vector space, and let S be a subset of V which spans V. Then S

   (a) Must have at most five elements

   (b) Must be linearly dependent

   (c) Must be linearly independent

   (d) Must be a basis for V

   (e) Must consist of at least five elements

   (f) Must have exactly five elements

   (g) Must have infinitely many elements

101. Let V be a five-dimensional vector space, and let S be a subset of V which is linearly independent. Then S

(a) Must have at most five elements
(b) Must span V
(c) Can have any number of elements (except zero)
(d) Must have infinitely many elements
(e) Must have exactly five elements
(f) Must be a basis for V
(g) Must consist of at least five elements

102. Let V be a five-dimensional vector space, and let S be a subset of V which is linearly dependent. Then S

(a) Must have at most five elements
(b) Must consist of at least five elements
(c) Must span V
(d) Must be a basis for V
(e) Must have exactly five elements
(f) Must have infinitely many elements
(g) Can have any number of elements (except zero)

103. Let V be a five-dimensional vector space, and let S be a subset of V which is a basis for V. Then S

(a) Must have at most five elements
(b) Must be linearly independent
(c) Can have any number of elements (except zero)
(d) Must span V
(e) Must be linearly dependent
(f) Must consist of at least five elements
(g) Must have exactly five elements

104. Let V be a five-dimensional vector space, and let S be a subset of V consisting of three vectors. Then S

(a) Must be linearly dependent, and must span V
(b) Cannot span V, but can be linearly independent or dependent
(c) Must be linearly dependent, but may or may not span$V$
(d) Must be linearly independent, but may or may not span V
(e) Must be linearly independent, but cannot span V
(f) Can span V, but only if it is linearly independent, and vice versa
(g) May or may not be linearly independent, and may or may not span V

105. Let V be a three-dimensional vector space, and let S be a subset of V consisting of five vectors. Then S

(a) Can span V,but only if it is linearly independent, and vice versa
(b) May or may not be linearly independent, and may or may not span V
(c) Must be linearly independent, but cannot span V
(d) Must be linearly dependent, and must span V
(e) Cannot span V, but can be linearly independent or dependent
(f) Must be linearly independent, but may or may not span V
(g) Must be linearly dependent, but may or may not span V

106. Let V be a five-dimensional vector space, and let S be a subset of V consisting of five vectors. Then S

(a) Must be linearly dependent, but may or may not span V
(b) Must be linearly dependent, and must span V
(c) Must be a basis of V
(d) Can span V, but only if it is linearly independent, and vice versa
(e) Cannot span V, but can be linearly independent or dependent
(f) Must be linearly independent, but may or may not span V
(g) Must be linearly independent, but cannot span V

107. If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5$ are five vectors in $\mathbf{R}^3$, then the number of redundant vectors

(a) Can be any number from zero to five
(b) Can be any number from two to five
(c) Can be any number from zero to three
(d) Can be any number from zero to two
(e) Must be two
(f) Must be zero
(g) Is three

108. If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are three vectors in $\mathbf{R}^5$, then the number of redundant vectors

(a) Can be any number from zero to three
(b) Can be any number from two to five
(c) Can be any number from zero to two
(d) Can be any number from zero to five
(e) Must be two
(f) Must be zero
(g) Is three

109. If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5$ are five vectors in $\mathbf{R}^3$, then the number of redundant vectors

(a) Can be any number from two to five
(b) Can be any number from zero to three
(c) Can be any number from zero to five
(d) Can be any number from zero to two
(e) Must be two
(f) Must be zero
(g) Is three

110. If $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are three vectors in $\mathbf{R}^5$, then the number of non-redundant vectors

(a) Can be any number from zero to three
(b) Can be any number from zero to two
(c) Can be any number from two to five
(d) Can be any number from zero to five
(e) Must be two
(f) Must be zero
(g) Is three

111. The nullity of a 3×5 matrix

(a) Can be any number from zero to three
(b) Can be any number from zero to two
(c) Can be any number from two to five
(d) Can be any number from zero to five
(e) Must be two
(f) Must be zero
(g) Is three

112. The nullity of a 5×3 matrix

(a) Can be any number from zero to three
(b) Can be any number from zero to two
(c) Can be any number from two to five
(d) Can be any number from zero to five
(e) Must be two
(f) Must be zero
(g) Is three

113. Let A be an invertible 5×5 matrix. Which of the following statements is *false*?

(a) The linear transformation associated to A must be both one-to-one and onto
(b) There must exist a 5×5 matrix B, such that $AB = BA = I$
(c) The rank of A must equal 5
(d) The reduced row echelon form of A must be the identity matrix
(e) For every vector b in $\mathbf{R}^5$, there must be exactly one solution to the equation $Ax = b$
(f) Every row of A must contain a leading 1
(g) The row-reduced echelon form A must contain no free variables