# Chapter 3

Directions: For questions 1 - 11 mark each statement True or False. Justify each answer.

1. (**True** | **False**) Asking whether the linear system corresponding to an augmented matrix $\left[\begin{array}{ccc|c} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{b} \end{array}\right]$ has a solution amounts to asking whether $\mathbf{b}$ is in Span $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$

2. (**True** | **False**) If the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent, then $\mathbf{b}$ is not n the set spanned by the columns of $A$

3. (**True** | **False**) The columns of matrix $A$ are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution

4. (**True** | **False**) If $S$ is a linearly dependent set, then each vector is a linear combination of the other vectors in $S$

5. (**True** | **False**) The columns of any $4 \times 5$ matrix are linearly dependent

6. (**True** | **False**) If x and y are linearly independent, and if $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, then $\mathbf{z}$ is in Span $\{\mathbf{x}, \mathbf{y}\}$

7. (**True** | **False**) If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent

8. (**True** | **False**) If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\mathbf{z}$ is in Span $\{\mathbf{x}, \mathbf{y}\}$, then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent

9. (**True** | **False**) If a set in $\mathbf{R}^n$ is linearly dependent, then the set contains more vectors than there are entries in each vector

10. (**True** | **False**) The columns of the standard matrix for a linear transformation from $\mathbf{R}^n$ to $\mathbf{R}^m$ are the images of the columns of the $n \times n$ identity matrix

11. (**True** | **False**) The standard matrix of a linear transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$ that reflects points through the horizontal axis, the vertical axis, or the origin has the form $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$, where $a$ and $d$ are $\pm 1$

    Directions: For questions 12 - 18, mark each statement True or False. Justify each answer. If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case.

12. (**True** | **False**) If $A$ is an $m \times n$ matrix and the equation is $A\mathbf{x} = \mathbf{b}$ is consistent for some $\mathbf{b}$, then the columns of $A$ span $\mathbf{R}^m$.

13. (**True** | **False**) If $A$ and $B$ are row equivalent $m \times n$ matrices and if the columns of $A$ span $\mathbf{R}^m$, then so do the columns of $B$.

14. (**True** | **False**) If none of the vectors in the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ in $\mathbf{R}^3$ is a multiple of one of the other vectors, the $S$ is linearly independent.

15. (**True** | **False**) If $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent, then $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are not in $\mathbf{R}^2$.

16. (**True** | **False**) In some cases, it is possible for four vectors to span $\mathbf{R}^5$.

17. (**True** | **False**) If $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbf{R}^m$, then $-\mathbf{u}$ is in Span$\{\mathbf{u}, \mathbf{v}\}$.

18. (**True** | **False**) Suppose that $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are in $\mathbf{R}^5$, $\mathbf{v}_2$ is not a multiple of $\mathbf{v}_1$, and $\mathbf{v}_3$ is not a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$, then $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent.

Directions: For questions 19 - 28, mark each statement True or False. Justify each answer.

19. (**True** | **False**) A subspace of $\mathbf{R}^n$ is any set $H$ such that (i) the zero vector is in $H$, (ii) $\mathbf{u}, \mathbf{v}$ and $\mathbf{u}+\mathbf{v}$ are in $H$, and (iii) $c$ is a scalar and $c\,\mathbf{u}$ is in $H$

20. (**True** | **False**) If $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are in $\mathbf{R}^n$, then Span $\{\mathbf{v}_1, \ldots, \mathbf{v}_2\}$ is the same as the column space of the matrix $\begin{bmatrix}\mathbf{v}_1 \cdots \mathbf{v}_p\end{bmatrix}$

21. (**True** | **False**) The set of all solutions of a system of $m$ homogeneous equations in $n$ unknowns is a subspace of $\mathbf{R}^m$

22. (**True** | **False**) The columns of an invertible $n \times n$ matrix form a basis for $\mathbf{R}^n$

23. (**True** | **False**) Row operations do not affect linear dependence relations among the columns of a matrix

24. (**True** | **False**) A subset $H$ of $\mathbf{R}^n$ is a subspace if the zero vector is in $H$

25. (**True** | **False**) Given vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$ in $\mathbf{R}^n$, the set of all linear combinations of these vectors is a subspace of $\mathbf{R}^n$

26. (**True** | **False**) The null space of an $m \times n$ matrix is a subspace of $\mathbf{R}^n$

27. (**True** | **False**) The column space of a matrix $A$ is the set of solutions of $A\,\mathbf{x} = \mathbf{b}$

28. (**True** | **False**) If $B$ is an echelon form of a matrix $A$, then the pivot columns of $B$ form a basis for Col $A$

Directions: For questions 29 - 38, $A$ is an $m \times n$ matrix. Mark each answer True or False. Justify your answer.

29. (**True** | **False**) If $\beta = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a basis for a subspace $H$ and if $\mathbf{x} = c_1\,\mathbf{v}_1 + \cdots + c_p\,\mathbf{v}_p$, then $c_1, \ldots, c_p$ are the coordinates of $\mathbf{x}$ relative to the basis $\beta$

30. (**True** | **False**) Each line in $\mathbf{R}^n$ is a one-dimensional subspace of $\mathbf{R}^n$

31. (**True** | **False**) The dimension of Col $A$ is the number of pivot columns of $A$

32. (**True** | **False**) The dimensions of Col $A$ and Nul $A$ add up to the number of columns of $A$

33. (**True** | **False**) If a set of $p$ vectors spans a $p$-dimensional subspace $H$ of $\mathbf{R}^n$, then these vectors form a basis for $H$

34. (**True** | **False**) If $B$ is a basis for a subspace $H$, then each vector in $H$ can be written in only one way as a linear combination of the vectors in $B$

35. (**True** | **False**) If $\beta = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ is a basis for a subspace $H$ of $\mathbf{R}^n$, then the correspondence $\mathbf{x} \mapsto [\mathbf{x}]_\beta$ makes $H$ look and act the same as $\mathbf{R}^p$

36. (**True** | **False**) The dimension of Nul A is the number of variables in the equation $A\mathbf{x} = \mathbf{0}$

37. (**True** | **False**) The dimension of the column space of $A$ is rank $A$

38. (**True** | **False**) If $H$ is a $p$-dimensional subspace of $\mathbf{R}^n$, then a linearly independent set of $p$ vectors in $H$ is a basis for $H$

Directions: For questions 39 - 47, mark each statement True or False. Justify each answer.

39. (**True** | **False**) If $\mathbf{f}$ is a function in the vector space $V$ of all real-valued functions on $\mathbf{R}$ and if $\mathbf{f}(t) = 0$ for some $t$, then  is the zero vector in $V$

40. (**True** | **False**) A vector is an arrow in three-dimensional space

41. (**True** | **False**) A subset $H$ of a vector space $V$ is a subspace of $V$ if the zero vector is in $H$

42. (**True** | **False**) A subspace is also a vector space

43. (**True** | **False**) A vector is any element of a vector space

44. (**True** | **False**) If $\mathbf{u}$ is a vector space in $V$, then $(-1)\,\mathbf{u}$ is the same as the negative of $\mathbf{u}$

45. (**True** | **False**) A vector space is also a subspace

46. (**True** | **False**) $\mathbf{R}^2$ is a subspace of $\mathbf{R}^3$

47. (**True** | **False**) A subset $H$ of a vector space $V$ is a subspace of $V$ if the following conditions are satisfied: (i) the zero vector of $V$ is in $H$, (ii) $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{u}+\mathbf{v}$ are in $H$, and (iii) $c$ is a scalar and $c\,\mathbf{u}$ is in $H$

Directions: For questions 48 - 58, $A$ denotes an $m \times n$ matrix. Mark each statement True or False. Justify your answer.

48. (**True** | **False**) The null space of $A$ is the solution set of the equation $A\mathbf{x} = \mathbf{0}$

49. (**True** | **False**) The null space of an $m \times n$ matrix is in $\mathbf{R}^m$

50. (**True** | **False**) The column space of $A$ is the range of the mapping $\mathbf{x} \mapsto A\mathbf{x}$

51. (**True** | **False**) If the equation $A\mathbf{x} = \mathbf{b}$ is consistent, then Col $A$ is $\mathbf{R}^m$

52. (**True** | **False**) The kernel of a linear transformation is a vector space

53. (**True** | **False**) Col $A$ is the set of all vectors that can be written as $A\mathbf{x}$ for some $\mathbf{x}$

54. (**True** | **False**) A null space is a vector space

55. (**True** | **False**) The column space of an $m \times n$ matrix is in $\mathbf{R}^m$

56. (**True** | **False**) Col $A$ is the set of all solutions of $A\mathbf{x} = \mathbf{b}$

57. (**True** | **False**) Nul $A$ is the kernel of the mapping $\mathbf{x} \mapsto A\mathbf{x}$

58. (**True** | **False**) The set of all solutions of a homogeneous linear differential equation is the kernel of a linear transformation

Directions: For questions 59 - 68, mark each statement True or False. Justify each answer.

59. (**True** | **False**) A single vector by itself is linearly dependent

60. (**True** | **False**) If $H = \text{Span}\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$, then $\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$ is a basis for $H$

61. (**True** | **False**) The columns of an invertible $n \times n$ matrix form a basis for $\mathbf{R}^n$

62. (**True** | **False**) A basis is a spanning set that is as large as possible

63. (**True** | **False**) In some cases, the linear dependence relations among the columns of a matrix can be affected by certain elementary row operations on the matrix

64. (**True** | **False**) A linearly independent set in a subspace $H$ is a basis for $H$

65. (**True** | **False**) If a finite set $S$ of nonzero vectors spans a vector space $V$,then some subset of $S$ is a basis for $V$

66. (**True** | **False**) A basis is a linearly independent set that is as large as possible

67. (**True** | **False**) The standard method for producing a spanning set for Nul $A$, sometimes fails to produce a basis for Nul $A$

68. (**True** | **False**) If $B$ is an echelon form of a matrix $A$, then the pivot columns of $B$ form a basis for Col $A$

For questions 69 - 82, mark each statement True or False

69. (**True** | **False**) The following vectors are linearly independent: $(1, 0, 0)$, $(0, 0, 2)$, $(3, 0, 4)$

70. (**True** | **False**) A set of 4 vectors from $R^3$ could be linearly independent

71. (**True** | **False**) A set of 2 vectors from $R^3$ must be linearly independent

72. (**True** | **False**) A set of 3 vectors from $R^3$ could be linearly independent

73. (**True** | **False**) A set of 5 vectors from $R^4$ could be linearly independent

74. (**True** | **False**) The row space of a matrix $A$ is the same as the column space of $A^T$

75. (**True** | **False**) The nonpivot columns of a matrix are always linearly dependent.

76. (**True** | **False**) Row operations in matrix $A$ can change the linear dependence relations among the columns of $A$.

77. (**True** | **False**) Row operations on matrix $A$ can change the linear dependence relations among the rows of $A$.

78. (**True** | **False**) Row operations on a matrix can change the null space.

79. (**True** | **False**) If an $m \times n$ matrix $A$ is row equivalent to an echelon matrix $U$ and if $U$ has $k$ nonzero rows, then the dimension of the solution space of $A\mathbf{x} = \mathbf{0}$ is $m - k$.

80. (**True** | **False**) The nonzero rows of a matrix $A$ form a basis for Row $A$.

81. (**True** | **False**) If matrices $A$ and $B$ have the same reduced echelon form, then Row $A$ = Row $B$.

82. (**True** | **False**) If $H$ is a subspace of $\mathbf{R}^3$, then there is a $3 \times 3$ matrix $A$ such that $H$ = Col $A$.

Directions: For questions 83 - 88, mark each statement True or False. Justify each answer. (If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case.) Assume $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are vectors in a nonzero finite-dimensional vector space $V$, and $S = \{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$.

83. (**True** | **False**) The set of all linear combinations of $\mathbf{v}_1, \ldots, \mathbf{v}_p$ is a vector space.

84. (**True** | **False**) If $\{\mathbf{v}_1, \ldots, \mathbf{v}_{p-1}\}$ spans $V$, then $S$ spans $V$.

85. (**True** | **False**) If $\{\mathbf{v}_1, \ldots, \mathbf{v}_{p-1}\}$ is linearly independent, then so is $S$.

86. (**True** | **False**) If $S$ is linearly independent, then $S$ is a basis for $V$.

87. (**True** | **False**) If Span $S = V$, then some subset of $S$ is a basis for $V$.

88. (**True** | **False**) If dim $V = p$ and Span $S = V$, then $S$ cannot be linearly dependent.