Chapter 4

Directions: For questions 1 - 6, mark each statement True or False. Justify each answer. Unless stated otherwise, B is a basis for a vector space V.

1. (**True** | **False**) If $\mathbf{x}$ is in V and if B contains n vectors, then the B-coordinate vector of $\mathbf{x}$ is in $\mathbf{R}^n$
2. (**True** | **False**) If P_B is the change-of-coordinates matrix, then $[\mathbf{x}]_B = P_B\,\mathbf{x}$, for $\mathbf{x}$ in V
3. (**True** | **False**) The vector spaces in $\mathbf{P}_3$ and $\mathbf{R}^3$ are isomorphic
4. (**True** | **False**) If B is the standard basis for $\mathbf{R}^n$, then the B-coordinate vector of an $\mathbf{x}$ in $\mathbf{R}^n$ is $\mathbf{x}$ itself
5. (**True** | **False**) The correspondence $[\mathbf{x}]_B \mapsto \mathbf{x}$ is called coordinate mapping
6. (**True** | **False**) In some cases, a plane in $\mathbf{R}^3$ can be isomorphic to $\mathbf{R}^2$

   Directions: For questions 7 - 15, V is a vector space. Mark each statement True or False. Justify each answer.

7. (**True** | **False**) The number of pivot columns of a matrix equals the dimension of its column space
8. (**True** | **False**) A plane in R^3 is a two-dimensional subspace of R^3
9. (**True** | **False**) If dim $V = n$ and S is is a linearly independent set in V, then S is a basis for V
10. (**True** | **False**) If a set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ spans a finite-dimensional vector space V and if T is a set of more than p vectors in V, then T is linearly dependent
11. (**True** | **False**) $\mathbf{R}^2$ is a two-dimensional subspace of $\mathbf{R}^3$
12. (**True** | **False**) The number of variables in the equation $A\mathbf{x} = \mathbf{0}$ equals the dimension of Nul A
13. (**True** | **False**) A vector space is infinite-dimensional if it is spanned by an infinite set
14. (**True** | **False**) If dim $V = n$ and if S spans V,then S is a basis of V
15. (**True** | **False**) The only three-dimensional subspace of $\mathbf{R}^3$ is $\mathbf{R}^3$ itself

    Directions: For questions 16 - 21, V is a nonzero finite-dimensional vector space, and the vectors listed belong to V. Mark each statement True or False. Justify each answer.

16. (**True** | **False**) If there exists a set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ that spans V, then dim $V \leq p$
17. (**True** | **False**) If there exists a linearly independent set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ in V, then dim $V \geq p$
18. (**True** | **False**) If dim $V = p$, then there exists a spanning set of $p + 1$ vectors in V
19. (**True** | **False**) If there exists a linearly dependent set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ in V, then dim $V \leq p$

20. (**True** | **False**) If every set of p elements in V fails to span V, then dim $V > p$

21. (**True** | **False**) If $p \geq 2$ and dim $V = p$, then every set of $p - 1$ nonzero vectors is linearly independent

Directions: For questions 22 - 30, A is an $m \times n$ matrix. Mark each statement True or False. Justify each answer.

22. (**True** | **False**) The row space of A is the same as the column space of A^T

23. (**True** | **False**) If B is any echelon form of A, and if B has three nonzero rows, then the first three rows of A form a basis for Row A

24. (**True** | **False**) The dimensions of the row space and the column space of A are the same, even if A is not square

25. (**True** | **False**) The sum of the dimensions of the row space and the null space of A equals the number of rows in A

26. (**True** | **False**) If B is any echelon form of A, then the pivot columns of B form a basis for the column space of A

27. (**True** | **False**) Row operations preserve the linear dependence relations among the rows of A

28. (**True** | **False**) The dimension of the null space of A is the number of columns of A that are *not* pivot columns

29. (**True** | **False**) The row space of A^T is the same as the column space of A

30. (**True** | **False**) If A and B are row equivalent, then their row spaces are the same

Directions: For questions 31 - 34, B and C are bases for a vector space V. Mark each statement True or False. Justify your answer.

31. (**True** | **False**) The columns of the change-of-coordinates matrix ${}_C P_B$ are B-coordinate vectors of the vectors in C

32. (**True** | **False**) If $V = \mathbf{R}^n$ and C is the standard basis for V, the ${}_C P_B$ is the same as the change-of-coordinates matrix I_B

33. (**True** | **False**) The columns of ${}_C P_B$ are linearly independent

34. (**True** | **False**) If $V = \mathbf{R}^2$, $B = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $C = \{\mathbf{c}_1, \mathbf{c}_2\}$, then row reduction of $\begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \mathbf{b}_1 & \mathbf{b}_2 \end{bmatrix}$ to $\begin{bmatrix} I & P \end{bmatrix}$ produces a matrix P that satisfies $[\mathbf{x}]_B = P[\mathbf{x}]_C$ for all $\mathbf{x}$ in V

Directions: For questions 35 - 38, mark each statement True or False. Justify each answer. (If true, cite appropriate facts or theorems. If false, explain why or give a counterexample that shows why the statement is not true in every case.)

35. (**True** | **False**) If A is $m \times n$ and rank $A = m$, then the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one.

36. (**True** | **False**) If A is $m \times n$ and the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ is onto, then rank $A = m$.

37. (**True** | **False**) A change-of-coordinates matrix is always invertible.

38. (**True** | **False**) If $X = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\}$ and $Y = \{\mathbf{y}_1, \ldots, \mathbf{y}_n\}$ are bases for a vector space V, then the jth column of the change-of-coordinates matrix ${}_Y I_X$ is the coordinate vector $K_X(\mathbf{y}_j)$.

For questions XX - XX, mark each statement True or False. Justify each answer.

39. (**True** | **False**) The function $h(t) = 4 + 3t$ is a linear combination of the functions $f(t) = (1+t)^2$ and $g(t) = 2 - t - 2t^2$

40. (**True** | **False**) The function $h(t) = \sin(t+2)$ is a linear combination of the functions $f(t) = \sin t$ and $g(t) = \cos t$.

41. (**True** | **False**) $h(t) = t^2$ is a linear combination of $f(t) = (1-t)^2$ and $g(t) = (1+t)^2$