Chapter 5

Directions: For questions 1 - 11, A and B are are $n \times n$ matrices. Mark each statement True or False. Justify each answer.

1. (**True** | **False**) An elementary row operation on A does not change the determinant
2. (**True** | **False**) A row replacement operation does not affect the determinant of a matrix
3. (**True** | **False**) The determinant of A is the product of the pivots in any echelon form U of A, multiplied by $(-1)^r$, where r is the number of row interchanges made during row reduction from A to U
4. (**True** | **False**) If the columns of A are linearly dependent, then det $A = 0$
5. (**True** | **False**) $\det(A+B) = \det(A) + \det(B)$
6. (**True** | **False**) If three row interchanges are made in succession, then the new determinant equals the old determinant
7. (**True** | **False**) The determinant of A is the product of the diagonal entries in A
8. (**True** | **False**) If det A is zero, then two rows or two columns are the same, or a row or a column is zero
9. (**True** | **False**) det $A^{-1} = (-1)$ det A
10. (**True** | **False**) $(\det A)(\det B) = \det AB$
11. (**True** | **False**) det $A^T = (-1)$ det A

Directions: For questions 12 - 26, mark each statement True or False. Justify each answer. Assume that all matrices here are square.

12. (**True** | **False**) If A is a 2×2 matrix with a zero determinant, then one column of A is a multiple of the other.
13. (**True** | **False**) If two rows of a 3×3 matrix A are the same, then det $A = 0$.
14. (**True** | **False**) If A is a 3×3 matrix, then det $5A = 5$ det A.
15. (**True** | **False**) If A and B are $n \times n$ matrices, with det $A = 2$ and det $B = 3$, then $\det(A+B) = 5$.
16. (**True** | **False**) If A is $n \times n$ and det $A = 2$, then det $A^3 = 6$.
17. (**True** | **False**) If B is produced by interchanging two rows of A, then det $B =$ det A.
18. (**True** | **False**) If B is produced by multiplying row 3 of A by 5, then det $B = 5 \cdot$ det A.
19. (**True** | **False**) If B is formed by adding to one row of A of a linear combination of the other rows, then det $B =$ det A.
20. (**True** | **False**) det $A^T = -$det A.

21. (**True** | **False**) $\det(-A) = -\det A$.

22. (**True** | **False**) $\det A^T A \geq 0$.

23. (**True** | **False**) If $A^3 = 0$, then $\det A = 0$.

24. (**True** | **False**) If A is invertible, then $\det A^{-1} = \det A$.

25. (**True** | **False**) If A is invertible, then $(\det A)(\det A^{-1}) = 1$.

26. (**True** | **False**) The determinant of a triangular matrix is the sum of the entries on the main diagonal

Directions: For questions 27 - 35, A is an $n \times n$ matrix. Mark each statement True or False. Justify each answer.

27. (**True** | **False**) If $A\mathbf{x} = \lambda\mathbf{x}$ for some vector $\mathbf{x}$, then λ is an eigenvalue of A

28. (**True** | **False**) A matrix A is not invertible if and only if 0 is an eigenvalue of A

29. (**True** | **False**) A number c is an eigenvalue of A if and only if the equation $(A - cI)\mathbf{x} = \mathbf{0}$ has a nontrivial solution

30. (**True** | **False**) Finding an eigenvector of A may be difficult, but checking whether a given vector is in fact an eigenvector is easy

31. (**True** | **False**) To find the eigenvalues of A, reduce A to echelon form

32. (**True** | **False**) If $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ, then $\mathbf{x}$ is an eigenvector of A

33. (**True** | **False**) If $\mathbf{v}_1$ and $\mathbf{v}_2$ are linearly independent eigenvectors, then they correspond to distinct eigenvalues

34. (**True** | **False**) The eigenvalues of a matrix are on its main diagonal

35. (**True** | **False**) An eigenspace of A is a null space of a certain matrix

Directions: For questions 36 - 45, A, B, P, and D are $n \times n$ matrices. Mark each statement True or False. Justify each answer.

36. (**True** | **False**) If $\lambda + 5$ is a factor of the characteristic polynomial of A, then 5 is an eigenvalue of A

37. (**True** | **False**) A row replacement operation on A does not change the eigenvalues

38. (**True** | **False**) A is diagonalizable if $A = PDP^{-1}$ for some matrix D and some invertible matrix P

39. (**True** | **False**) If $\mathbf{R}^n$ has a basis of eigenvectors of A, then A is diagonalizable

40. (**True** | **False**) A is diagonalizable if and only if A has n eigenvalues, counting multiplicities

41. (**True** | **False**) If A is diagonalizable, then A is invertible

42. (**True** | **False**) A is diagonalizable if A has n eigenvectors

43. (**True** | **False**) If A is diagonalizable, then A has n distinct eigenvalues

44. (**True** | **False**) If $AP = PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A

45. (**True** | **False**) If A is invertible, then A is diagonalizable

Directions: For questions 46 - 65, mark each statement True or False. Justify each answer. Let A and B represent square matrices of appropriate sizes.

46. (**True** | **False**) If A is invertible and 1 is an eigenvalue of A, then 1 is also an eigenvalue of A^{-1}.

47. (**True** | **False**) If A is row equivalent to the identity matrix I, then A is diagonalizable.

48. (**True** | **False**) If A contains a row or column of zeros, then 0 is an eigenvalue of A.

49. (**True** | **False**) Each eigenvalue of A is also an eigenvalue of A^2.

50. (**True** | **False**) Each eigenvector of A is also an eigenvector A^2.

51. (**True** | **False**) Each eigenvector of an invertible matrix A is also an eigenvector of A^{-1}.

52. (**True** | **False**) Eigenvalues must be nonzero scalars.

53. (**True** | **False**) Eigenvectors must be nonzero vectors.

54. (**True** | **False**) Two eigenvectors corresponding to the same eigenvalue are always linearly dependent.

55. (**True** | **False**) The sum of two eigenvectors of a matrix A is also an eigenvector of A.

56. (**True** | **False**) The eigenvalues of an upper triangular matrix A are exactly the nonzero entries on the diagonal of A.

57. (**True** | **False**) The matrices A and A^T have the same eigenvalues, counting multiplicities.

58. (**True** | **False**) If a 5×5 matrix A has fewer than 5 distinct eigenvalues, then A is not diagonalizable.

59. (**True** | **False**) There exists a 2×2 matrix that has no eigenvectors in $\mathbf{R}^2$.

60. (**True** | **False**) If A is diagonalizable, then the columns of A are linearly independent.

61. (**True** | **False**) A nonzero vector cannot correspond to two different eigenvalues of A.

62. (**True** | **False**) A (square) matrix A is invertible if and only if there is a coordinate system in which the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is represented by a diagonal matrix.

63. (**True** | **False**) If each vector $\mathbf{e}_j$ in the standard basis for $\mathbf{R}^n$ is an eigenvector of A, then A is a diagonal matrix.

64. (**True** | **False**) An $n \times n$ matrix with n linearly independent eigenvectors is invertible.

65. (**True** | **False**) If A is an $n \times n$ diagonalizable matrix, then each vector in $\mathbf{R}^n$ can be written as a linear combination of eigenvectors of A.

For questions 66 - 70, mark each statement True or False.

66. (**True** | **False**) $\det(A+B) = \det A + \det B$.

67. (**True** | **False**) $\det(AB) = \det A \det B$.

68. (**True** | **False**) Let A be an $n \times n$ matrix. The determinant of A is the product of the diagonal entries in A

69. (**True** | **False**) Any nonzero linear combination of two eigenvectors of a matrix A is an eigenvector of A

70. (**True** | **False**) Invertible matrices are diagonalizable