Chapter 6

1. Which of the following is an orthonormal basis for the column space of the real matrix $\begin{bmatrix} 1 & -1 & 2 & -3 \\ -1 & 1 & -3 & 2 \\ 2 & -2 & 5 & -5 \end{bmatrix}$?

   (a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

   (b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

   (c) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \\ 0 \end{bmatrix} \right\}$

   (d) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix} \right\}$

   (e) $\left\{ \begin{bmatrix} \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{bmatrix} \right\}$

2. Let $\mathbf{i} = (1,0,0)$, $\mathbf{j} = (0,1,0)$, and $\mathbf{k} = (0,0,1)$. The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal if $v_1 = \mathbf{i}+\mathbf{j}-\mathbf{k}$ and $\mathbf{v}_2 =$

   (a) $\mathbf{i}+\mathbf{j}-\mathbf{k}$
   (b) $\mathbf{i}-\mathbf{j}+\mathbf{k}$
   (c) $\mathbf{i}+\mathbf{k}$
   (d) $\mathbf{j}-\mathbf{k}$
   (e) $\mathbf{i}+\mathbf{j}$

3. An inner product on a real vector space is a map

   (a) $\langle\ ,\ \rangle : V \times V \to \mathbf{R}$
   (b) $\langle\ ,\ \rangle : V \times V \to V$
   (c) $\langle\ ,\ \rangle : \mathbf{R} \times V \to V$

4. Which of the following statements is (or are) correct?

   (a) If $\langle\ ,\ \rangle : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}$ is an inner product on the real vector space $\mathbf{R}^n$, then $\langle x, y\rangle = x_1y_1+\cdots+x_ny_n$ for all $x, y \in \mathbf{R}^n$

(b) If $\langle\, ,\, \rangle : \mathbf{R}^n \times \mathbf{R}^n \to \mathbf{R}^n$ is an inner product on the real vector space $\mathbf{R}^n$, then $\langle x,\, y\rangle = (x_1y_1, \ldots, x_ny_n)$ for all $x, y \in \mathbf{R}^n$

(c) If one defines $\langle x,\, y\rangle = x_1y_1 + \cdots + x_ny_n$ for all $x, y \in \mathbf{R}^n$, then one obtains an inner product on $\mathbf{R}^n$

5. Let $V = \mathbf{R}^2$ with the standard inner product. Which of the following tuples of elements of V forms an orthonormal basis?

(a) ((1, −1), (−1, −1))

(b) ((−1,0), (0, −1))

(c) ((1,0), (0,1), (1,1))

6. For which subspaces $U \subset V$ is the orthogonal projection $P_U : V \to U$ an orthogonal map?

(a) for each U

(b) only for $U = V$

(c) only for $V = \{0\}$

7. Which of the following matrices is (or are) orthogonal?

(a) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

8. For what pair of real numbers (a_1, a_2, a_3), does the function $\left\langle \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right\rangle = a_1x_1y_3 + a_2x_2y_2 + a_3x_3y_1$ define an inner product on $\mathbf{R}^3$?

(a) $(1, 0, -1)$

(b) $(1, -1, 1)$

(c) $(-1, 2, 1)$

(d) $(1, 2, 1)$

(e) None of the above

9. The set of vectors $\{(1,1,1), (1,-1,0), (1,1,-2)\}$ is an orthogonal basis of $\mathbf{R}^3$. Find (c_1, c_2, c_3) such that $(1, 0, 0) = c_1(1, 1, 1) + c_2(1, -1, 0) + c_3(1, 1, -2)$

(a) $(-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{6})$

(b) $(\frac{1}{2}, -\frac{1}{3}, \frac{1}{6})$

(c) $(-\frac{1}{3}, -\frac{1}{2}, -\frac{1}{6})$

(d) $(\frac{1}{9}, \frac{1}{4}, \frac{1}{36})$
(e) $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$
(f) $(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}})$

10. If A and B are non-zero square matrices, then $AB = 0$ implies

    (a) A and B are orthogonal

    (b) A and B are singular

    (c) B is singular

    (d) A is singular

11. What is the dot product of $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$?

    (a) $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$

    (b) 5

    (c) 0

    (d) The dot product cannot be computed for these vectors

12. What is the dot product of $\begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$?

    (a) $\begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$

    (b) -4

    (c) 0

    (d) The dot product cannot be computed for these vectors

13. The magnitude of a vector v is defined to be its dot product with itself $v \cdot v$. What is the magnitude of the vector $(2, -1, -1)$?

    (a) 0

    (b) 2

    (c) 4

    (d) 6

14. Is it possible for a vector to have a negative magnitude?

(a) Yes
(b) No
(c) Not enough information given

15. What can we say about two vectors whose dot product is negative?

    (a) The vectors are orthogonal
    (b) The angle between the two vectors is less than $90°$
    (c) The angle between the two vectors is greater than $90°$

16. Which of the following sets of vectors is *not* an orthogonal set?

    (a) (1,1,1),(1,0,−1)
    (b) (2,3),(−6,4)
    (c) (3,0,0,2),(0,1,0,1)
    (d) (0,2,0),(−1,0,3)
    (e) $(\cos\theta, \sin\theta), (\sin\theta, -\cos\theta)$

17. Let A be a square matrix whose columns are mutually orthogonal, nonzero vectors. Which of the following are true?

    (a)
    (b)
    (c)
    (d)
    (e)

18. Let A be any matrix. Which of the following are true?

    (a) The row space of A and the nullspace of A are orthogonal to each other
    (b) The column space of A and the row space of A are orthogonal to each other
    (c) The column space of A and the nullspace of A are orthogonal to each other
    (d) Exactly two of (a), (b), and (c) are true
    (e) All of (a), (b), and (c) are true

19. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the row space of A?

    (a) $(1, 1, -1)$
    (b) $(1, 4, 2)$
    (c) $(0, 0, 5)$

(d) $(-1, 0, 1)$

20. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the columns space of A?

    (a) $(1, 1, -1)$
    (b) $(1, 4, 2)$
    (c) $(0, 1, -2)$
    (d) $(2, 0, 2)$

21. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$. Which of the following vectors is orthogonal to the nullspace of A?

    (a) $(1, 1, -1)$
    (b) $(1, 4, 2)$
    (c) $(0, 1, -2)$
    (d) $(2, 0, 2)$

22. Which of the following sets of vetors is an orthonormal set?

    (a) $(1, 1, 1), (1, 0, -1)$
    (b) $(2, 3), (-6, 4)$
    (c) $(0, 2, 0), (-1, 0, 3)$
    (d) $(\cos\theta, \sin\theta), (\sin\theta, -\cos\theta)$

23. If $b = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and $y = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, then the orthogonal projection of b onto y is

    (a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$
    (b) $\begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$
    (c) $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$
    (d) $\begin{bmatrix} 1/10 \\ 3/10 \end{bmatrix}$

24. Let A be an $n \times p$ matrix. Let W be the column space of A, so W is a subspace of R^n. Let $b \in R^n$ and let z be an orthogonal projection of b on W. Then which of the following is *not* true?

    (a) $A^T(b - z) = 0$

(b) z is orthogonal to W

(c) $b - z$ is orthogonal to W

(d) z is the vector in W closest to b

25. Let $v_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, and v = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$. Let z be the orthogonal projection of v on the span of $\{v_1, v_2\}$, and let $A = \begin{bmatrix} v_1 & v_2 \end{bmatrix}$ Which of the following are true?

(a) $z = Ax$ for some x

(b) z is a linear combination of v_1 and v_2

(c) $z = -\frac{1}{5} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + \frac{7}{30} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$

(d) All of the above statements are true

(e) Exactly two of the above statements are true

(f) None of the above statements are true

26. Let A be an $m \times n$ matrix with linearly independent columns $x_1, x_2, \ldots, x_n$. Applying the Gram Schmidt process to $x_1, x_2, \ldots, x_n$ will produce

(a) an orthogonal basis for A

(b) an orthogonal basis for the column space of A

(c) an orthogonal basis for the row space of A

(d) an orthogonal basis for the null space of A

27. Let $v_1 = (2, -1, 0)$ and $v_2 = (1, 1, 1)$. The Gram-Schmidt process, when applied to these vectors, produces $\{v_1', v_2'\}$ where

(a) $v_1' = (2, -1, 0)$ and $v_2' = (-1, 2, 1)$

(b) $v_1' = (2, -1, 0)$ and $v_2' = (\frac{3}{5}, \frac{6}{5}, 1)$

(c) $v_1' = (2, -1, 0)$ and $v_2' = (\frac{2}{5}, \frac{-1}{5}, 0)$

(d) $v_1' = (2, -1, 0)$ and $v_2' = (\frac{7}{5}, \frac{6}{5}, 1)$

(e) $v_1' = (2, -1, 0)$ and $v_2' = (\frac{3}{2}, 3, 1)$

28. If $b = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ and l is the line $y = \frac{1}{2}x$, then the orthogonal projection of b onto l is

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$

(c) $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

(d) $\begin{bmatrix} 1/10 \\ 3/10 \end{bmatrix}$