Chapter 7

1. If A is an $n \times n$ real symmetric matrix, then which of the following is true?
   (a) Each eigenvalue of A is real
   (b) If A is invertible, then its inverse is also symmetric
   (c) If $Ax = 2x$ and $Ay = 3y$ then $x \cdot y = 0$
   (d) If λ_1 and λ_2 are two different eigenvalues of A and W_1 and W_2 are the corresponding eigenspaces, then W_1 and W_2 are orthogonal sets
   (e) All of the above are true
   (f) More than one, but not all, of the above are true

2. Which of the following matrices is symmetric?
   (a) $\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \\ 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \end{bmatrix}$
   (b) $\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \\ 1 & 3 & 0 & 0 \\ 2 & 4 & 0 & 0 \end{bmatrix}$
   (c) $\begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 3 & 1 \end{bmatrix}$

3. For a real symmetric matrix A to carry out the principal axes transformation means finding
   (a) A symmetric matrix P so that $P^{-1}AP$ is diagonal
   (b) An orthogonal matrix $P \in O(n)$ so that $P^{-1}AP$ is diagonal
   (c) An invertible matrix $P \in GL(n, \mathbf{R})$ so that $P^{-1}AP$ is diagonal

4. If a symmetric real $n \times n$ matrix A has only one eigenvector λ, then
   (a) A is already diagonal
   (b) a_{ij} for all $i, j = 1, \ldots, n$
   (c) $n = 1$

5. If A is a symmetric matrix, then $A^T =$
   (a) A
   (b) $|A|$
   (c) 0
   (d) The diagonal matrix