Chapter 7

Directions: For questions 1 - 7, mark each statement True or False. Justify each answer.

1. (**True** | **False**) An $n \times n$ matrix that is orthogonally diagonalizable must be symmetric
2. (**True** | **False**) If $A^T = A$ and if vectors $\mathbf{u}$ and $\mathbf{v}$ satisfy $A\,\mathbf{u} = 3\,\mathbf{u}$ and $A\,\mathbf{v} = 4\,\mathbf{v}$, then $\mathbf{u} \cdot \mathbf{v} = 0$
3. (**True** | **False**) An $n \times n$ symmetric matrix has n distinct real eigenvalues
4. (**True** | **False**) There are symmetric matrices that are not orthogonally diagonalizable
5. (**True** | **False**) If $B = PDP^T$, where $P^T = P^{-1}$ and D is is a diagonal matrix, then B is a symmetric matrix
6. (**True** | **False**) The dimension of an eigenspace of a symmetric matrix is sometimes less than the multiplicity of the corresponding eigenvalue
7. (**True** | **False**) If A is a symmetric, invertible matrix, then $A^{-1} = A^T$

Directions: For questions 8 - 18, matrices are $n \times n$ and vectors are in $\mathbf{R}^n$. Mark each statement True or False. Justify each answer.

8. (**True** | **False**) The matrix of a quadratic form is a symmetric matrix
9. (**True** | **False**) A quadratic form has no cross-product terms if and only if the matrix of the quadratic form is a diagonal matrix
10. (**True** | **False**) The principal axes of a quadratic form $\mathbf{x}^T\,A\,\mathbf{x}$ are eigenvectors of A
11. (**True** | **False**) A positive definite quadratic form Q satisfies $Q(\mathbf{x}) > 0$ for all $\mathbf{x}$ in $\mathbf{R}^n$
12. (**True** | **False**) If the eigenvalues of a symmetric matrix A are all positive, then the quadratic form $\mathbf{x}^T\,A\,\mathbf{x}$ is positive definite
13. (**True** | **False**) A Cholesky factorization of a symmetric matrix A has the form $A = R^T R$, for an upper triangular matrix R with positive diagonal entries
14. (**True** | **False**) The expression $||\,\mathbf{x}\,||^2$ is not a quadratic form
15. (**True** | **False**) If A is symmetric and P is an orthogonal matrix, then the change of variable $\mathbf{x} = P\,\mathbf{y}$ transforms $\mathbf{x}^T\,A\,\mathbf{x}$ into a quadratic form with no cross-product term
16. (**True** | **False**) If A is a 2×2 symmetric matrix, then the set of $\mathbf{x}$ such that $\mathbf{x}^T\,A\,\mathbf{x} = c$ (for a constant c) corresponds to either a circle, an ellipse, or a hyperbola
17. (**True** | **False**) An indefinite quadratic form is neither positive semidefinite nor negative semidefinite
18. (**True** | **False**) If A is symmetric and the quadratic form $\mathbf{x}^T\,A\,\mathbf{x}$ has only negative values for $\mathbf{x} \neq \mathbf{0}$, then the eigenvalues of A are all positive

Directions: For questions 19 - 32, mark each statement True or False. Justify each answer. In each part, A represents an $n \times n$ matrix.

19. (**True** | **False**) If A is orthogonally diagonalizable, then A is symmetric.

20. (**True** | **False**) If A is an orthogonal matrix, then A is symmetric.

21. (**True** | **False**) If A is an orthogonal matrix, then $||A\,\mathbf{x}\,|| = ||\,\mathbf{x}\,||$ for all $\mathbf{x}$ in $\mathbf{R}^n$.

22. (**True** | **False**) The principal axes of a quadratic form $\mathbf{x}^T\,A\,\mathbf{x}$ can be the columns of any matrix P that diagonalizes A.

23. (**True** | **False**) By a suitable change of variable, any quadratic form can be changed into one with no cross-product term.

24. (**True** | **False**) The largest value of a quadratic form $\mathbf{x}^T\,A\,\mathbf{x}$, for $||\,\mathbf{x}\,|| = 1$, is the largest entry on the diagonal of A.

25. (**True** | **False**) The maximum value of a positive definite quadratic form $\mathbf{x}^T\,A\,\mathbf{x}$ is the greatest eigenvalue of A.

26. (**True** | **False**) A positive definite quadratic form can be changed into a negative definite form by a suitable change of variable $\mathbf{x} = P\,\mathbf{u}$, for some orthogonal matrix P.

27. (**True** | **False**) An indefinite quadratic form is one whose eigenvalues are not definite.

28. (**True** | **False**) If P is an $n \times n$ orthogonal matrix, then the change of variable $\mathbf{x} = P\,\mathbf{u}$ transforms $\mathbf{x}^T\,A\,\mathbf{x}$ into a quadratic form whose matrix is $P^{-1}AP$.

29. (**True** | **False**) If U is $m \times n$ with orthogonal columns, then $UU^T\,\mathbf{x}$ is the orthogonal projection of $\mathbf{x}$ onto Col U.

30. (**True** | **False**) If B is $m \times n$ and $\mathbf{x}$ is a unit vector in $\mathbf{R}^n$, then $||B\,\mathbf{x}\,|| \leq \sigma_1$, where σ_1 is the first singular value of B.

31. (**True** | **False**) A singular value decomposition of an $m \times n$ matrix B can be written as $B = P\sum Q$, where P is an $m \times m$ orthogonal matrix, Q is an $n \times n$ orthogonal matrix, and $\sum$ is an $m \times n$ "diagonal" matrix.

32. (**True** | **False**) If A is $n \times n$, then A and A^TA have the same singular values.

Directions: For questions 33 - 36, mark each statement True or False. Justify each answer. Let A and B represent square matrices of appropriate sizes.

33. (**True** | **False**) If A and B are invertible $n \times n$ matrices, then AB is similar to BA.

34. (**True** | **False**) Similar matrices always have exactly the same eigenvalues.

35. (**True** | **False**) Similar matrices always have exactly the same eigenvectors.

36. (**True** | **False**) If A is similar to a diagonalizable matrix B, then A is also diagonalizable.