LINEAR ALGEBRA QUESTION BANK

(1) (12 points total) Circle True or False:

TRUE / FALSE: If A is any $n \times n$ matrix, and I_n is the $n \times n$ identity matrix, then $I_n A = A I_n = A$.

TRUE / FALSE: If A, B are $n \times n$ matrices, then the inverse of AB is $A^{-1}B^{-1}$.

TRUE / FALSE: If A, B are $n \times n$ matrices, then $(A+B)^{-1} = A^{-1} + B^{-1}$. (Hint: is this true for numbers?)

TRUE / FALSE: The Reduced Row Echelon Form of a matrix is unique.

TRUE / FALSE: If A and B are both 2×3 matrices, then their product AB is defined.

TRUE / FALSE: If A and B are both 2×3 matrices, then the product AB^T is defined.

(2) True or false: If a system of equations has more than one solution, it has infinitely many solutions.

(a) True
(b) False

(3) True or false: If a system of equations is consistent, then it cannot have any free variables.

(a) True
(b) False

(4) True or false: Let A be a 2×3 matrix. Then $Nul(A)$ is a subspace of $\mathbb{R}^2$.

(a) True
(b) False

(5) True or false: Let A be a 2×3 matrix. Then $Col(A)$ is a subspace of $\mathbb{R}^2$.

(a) True
(b) False

(6) True or false: If V is a vector space of dimension d, and $\{\mathbf{v}_1, \ldots, \mathbf{v}_d\}$ are d different vectors in V, then they must form a basis.

(a) True
(b) False

(7) True or false: If V is a subspace of $\mathbb{R}^n$, then every basis for V must have the same number of vectors.

(a) True
(b) False

(8) True or false: If V is a vector space of dimension d, and $\{\mathbf{v}_1, \ldots, \mathbf{v}_d\}$ are d linearly independent vectors in V, then they must span V.

(a) True
(b) False

(9) What is the dimension of the null space $\text{Nul}(A)$ of $A = \begin{bmatrix} 2 & 3 & 1 & -1 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$?

$A.$	1
$B.$	2
$C.$	$\boxed{3}$
$D.$	5

(10) What is the dimension of the column space $\text{Col}(A)$ of $A = \begin{bmatrix} 2 & 3 & 1 & -1 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$?

$A.$	1
$B.$	$\boxed{2}$
$C.$	3
$D.$	5

(11) What is the dimension of the left null space $\text{Nul}(A^T)$ of $A = \begin{bmatrix} 2 & 3 & 1 & -1 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$?

$A.$	$\boxed{1}$
$B.$	2
$C.$	3
$D.$	5

(12) What is the dimension of the row space $\mathrm{Col}(A^T)$ of $A = \begin{bmatrix} 2 & 3 & 1 & -1 & 0 \\ 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$?

$A.$ 1
$B.$ $\boxed{2}$
$C.$ 3
$D.$ 5

For questions 5 and 6: Suppose

$$A = \begin{bmatrix} 1 & -3 & 1 & -2 \\ 0 & 1 & -1 & 1 \\ 1 & -1 & -1 & 0 \end{bmatrix}$$

and its reduced echelon form is

$$U = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(13) Which of these is a basis for Col(A)?

A. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

C. $\boxed{\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ -1 \end{bmatrix} \right\}}$

D. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} \right\}$

(14) Which of these is a basis for Col(A^T)?

A. $\text{Span} \left\{ \begin{bmatrix} 1 \\ -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \end{bmatrix} \right\}$

B. $\text{Span} \left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ -3 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \end{bmatrix} \right\}$

D. $\boxed{\left\{ \begin{bmatrix} 1 \\ 0 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} \right\}}$

(15) The matrix for a 90° counterclockwise rotation in the x-y plane is

A. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

C. $\boxed{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}$

D. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(16) Let L be the linear transformation from $\mathbb{P}_2$ to $\mathbb{P}_2$ given by

$$L(p(t)) = 2p'(t) + 3p(t)$$

and let $\mathcal{B} = \{1, t, t^2\}$ be the standard basis for $\mathbb{P}_2$. Then the coordinate matrix A representing L with input and output basis $\mathcal{B}$ is:

A. $\begin{bmatrix} 3 & 0 & 0 \\ 2 & 3 & 0 \\ 0 & 4 & 3 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 3 & 0 \\ 0 & 2 & 6 \\ 0 & 0 & 2 \end{bmatrix}$

C. $\boxed{\begin{bmatrix} 3 & 2 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}}$

D. $\begin{bmatrix} 2 & 0 & 0 \\ 3 & 2 & 0 \\ 0 & 6 & 2 \end{bmatrix}$

(17) For every $m \times n$ matrix A, the orthogonal complement of $\mathrm{Col}(A)$ in $\mathbb{R}^m$ is $\mathrm{Nul}(A)$.
A. True
B. $\boxed{False}$

(18) For every $m \times n$ matrix A, the sum of the dimensions of $\mathrm{Nul}(A^T)$ and $\mathrm{Col}(A)$ is equal to m.
A. $\boxed{True}$
B. False

(19) If V is a 6-dimensional vector space, and $\mathbf{v}_1, \ldots, \mathbf{v}_m$ is a basis for V, then m must be equal to 6.
A. $\boxed{True}$
B. False

(20) If V is a 6-dimensional vector space, and $\mathbf{v}_1, \ldots, \mathbf{v}_6$ are six vectors in V, then they must form a basis of V.
A. True
B. $\boxed{False}$

(21) If V is a 6-dimensional subspace of $\mathbb{R}^{10}$, then the orthogonal complement $V^\perp$ must be 4-dimensional.
A. $\boxed{True}$
B. False

(22) If V is a 3-dimensional subspace of $\mathbb{R}^7$, and $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ are three linearly independent vectors in V, then they also span V.
A. $\boxed{True}$
B. False

(23) If V and W are subspaces of $\mathbb{R}^n$, and $W^\perp = V$, then $V^\perp = W$.
A. $\boxed{True}$
B. False

(24) Suppose $A = [\mathbf{a}_1 \dots \mathbf{a}_4]$ and $B = [\mathbf{b}_1 \dots \mathbf{b}_4]$ are two 4×4 matrices so that

$$AB = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 2 & 4 & 6 \\ 1 & 3 & 6 & 9 \\ 1 & 4 & 8 & 12 \end{bmatrix}.$$

What is $A\mathbf{b}_2$? That is, what is A times the second column of B?

(a) $\begin{bmatrix} 1 \\ 2 \\ 4 \\ 6 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ (c) Not enough information to tell.

(25) Let

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 5 & 2 \\ 6 & 1 & 2 \end{bmatrix}.$$

For which permutation matrix P does PA have an LU decomposition?

(a) $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(26) Suppose A is a matrix with LU decomposition:

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

If $\mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, the LU method for $A\mathbf{x} = \mathbf{b}$ gives

(a) $\mathbf{c} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$.

(b) $\mathbf{c} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$.

(c) $\mathbf{c} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$.

(d) $\mathbf{c} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$.

(27) What is the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}?$$

(a) $A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (c) $A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

(28) Suppose A is a 3×3 matrix so that

$$A \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad A \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \text{ and } \quad A \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

What is the first column of A^{-1}?

(a) $\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix}$ (e) Not enough information to tell

(29) Suppose A and B are invertible 3×3 matrices, with inverses

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

What is $(AB)^{-1}$?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -10 & -5 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -10 & 5 & 1 \end{bmatrix}$

(30) Which of the following are subspaces of $\mathbb{P}_2$, the vector space of polynomials with degree at most 2:
- $W_1 = \{a_0 + a_1 t + a_2 t^2 : a_0 = 1, \text{ and } a_1, a_2 \in \mathbb{R}\}$
- $W_2 = \{a_0 + a_1 t + a_2 t^2 : a_1 = 1, \text{ and } a_0, a_2 \in \mathbb{R}\}$
- $W_3 = \{a_0 + a_1 t + a_2 t^2 : a_1 = 0, \text{ and } a_0, a_2 \in \mathbb{R}\}$
- $W_4 = \{at + b(t-1) : a, b \in \mathbb{R}\}$

(a) W_3 only
(b) W_4 only
(c) W_3 and W_4 only
(d) W_1 and W_2 only
(e) All four are subspaces

(31) Which of the following are subspaces of the indicated vector space?
- $W_1 = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a - 2b = c, 4a + 2c = 1 \right\} \subseteq \mathbb{R}^3$

- $W_2 = \left\{ \begin{bmatrix} a-b \\ c \\ a+c \\ a-2b-c \end{bmatrix} : a, b, c \in \mathbb{R} \right\} \subseteq \mathbb{R}^4$
- $W_3 = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} : a \cdot b \geq 0 \right\} \subseteq \mathbb{R}^2$
- $W_4 = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} : a^2 + b^2 \leq 1 \right\} \subseteq \mathbb{R}^2$

(a) W_2 only
(b) W_1 and W_2 only
(c) W_2 and W_3 only
(d) All four are subspaces

(32) Suppose

$$A = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

are 3×4 matrices, and $\mathbf{b}$ is a vector in both $Col(A)$ and $Col(B)$. Suppose also that

$$Nul(A) = \{\mathbf{0}\} \quad \text{and} \quad Nul(B) = span \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Which of the following is true?

(a) $A\mathbf{x} = \mathbf{b}$ has a unique solution, but $B\mathbf{x} = \mathbf{b}$ does not.
(b) $B\mathbf{x} = \mathbf{b}$ has a unique solution, but $A\mathbf{x} = \mathbf{b}$ does not.
(c) Both $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{b}$ have unique solutions.
(d) Neither $A\mathbf{x} = \mathbf{b}$ nor $A\mathbf{x} = \mathbf{b}$ have unique solutions.

(33) Let

$$A = \begin{bmatrix} 1 & 3 \\ -2 & -6 \end{bmatrix}$$

Which of the following are in $Col(A)$?

- $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\mathbf{v}_2 = \begin{bmatrix} 4 \\ -12 \end{bmatrix}$
- $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- $\mathbf{v}_4 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

(a) $\mathbf{v}_4$ only
(b) $\mathbf{v}_1$ and $\mathbf{v}_4$ only

(c) $\mathbf{v}_2$ and $\mathbf{v}_4$ only
(d) $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_4$ only
(e) $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$

(34) Which of the following sets of vectors are linearly independent?

- $A = \left\{ \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix} \right\}$
- $B = \left\{ \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}$
- $C = \left\{ \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$
- $D = \left\{ \begin{bmatrix} -1 \\ 0 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ -5 \\ 0 \\ 10 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \\ -11 \end{bmatrix} \right\}$

(a) None of them are linearly independent
(b) D only
(c) A and D only
(d) B and D only
(e) C and D only

(35) True or false: If the columns of a matrix A are linearly independent, then the rows of A must also be linearly independent.

(a) True
(b) False

(36) True or false: The dimension of $Nul(A)$ must be equal to the number of zero rows at the bottom of an echelon form of A.

(a) True
(b) False

(37) True or false: For every matrix A, with echelon form U, the row space $Col(A^T)$ must be equal to the row space $Col(U^T)$.

(a) True
(b) False

(38) True or false: If the columns of a matrix A are linearly independent, then $Nul(A)$ must be $\{\mathbf{0}\}$.

(a) True

(b) False

(39) True or false: For every matrix A, the column space $Col(A)$ and null space $Nul(A)$ are orthogonal complements.

(a) True
(b) False

(40) True or false: For every matrix A, the row space $Col(A^T)$ and the null space $Nul(A)$ are orthogonal complements.

(a) True
(b) False

(41) True or false: Every orthonormal basis is an orthogonal basis.

(a) True
(b) False

(42) True or false: If $\mathcal{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ is any basis for $\mathbb{R}^n$, and $\mathbf{w}$ is another vector, then the projections of $\mathbf{w}$ onto each of the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ must sum back to $\mathbf{w}$.

(a) True
(b) False

(43) True of false: If $\mathcal{A}$ and $\mathcal{B}$ are two bases for $\mathbb{R}^n$, and $I_{\mathcal{B}\mathcal{A}}$ is the change of basis matrix for input basis $\mathcal{A}$ and output basis $\mathcal{B}$, then

$$\mathbf{v}_{\mathcal{B}} = I_{\mathcal{B}\mathcal{A}}\mathbf{v}_{\mathcal{A}}$$

for any vector $\mathbf{v} \in \mathbb{R}^n$.

(a) True
(b) False

(44) True or False: If a square matrix A has an eigenbasis, then A must be invertible.

(a) True
(b) False

(45) True or False: For all square matrices A and B, $\det(A + B) = \det(A) + \det(B)$.

(a) True

(b) False

(46) True or False: If Q is an orthogonal matrix, then $\det(Q)$ must be equal to 1.

(a) True
(b) False

(47) True or False: If A has an **orthogonal** basis of eigenvectors, then A must be symmetric.

(a) True
(b) False

(48) True or False: In a discrete dynamical system with transition matrix A, if all eigenvalues of A have absolute value smaller than 1, then $\lim_{n\to\infty} \mathbf{v}_n = \mathbf{0}$ for every orbit $\mathbf{v}_0$, $\mathbf{v}_1$, $\mathbf{v}_2$, ... (You may assume that A has a basis of eigenvectors.)

(a) True
(b) False

(49) The determinant of the matrix $A = \begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & d & 1 \end{bmatrix}$ is:

A.	$abcd$
B.	a
C.	b
D.	$-c$
E.	c

(50) The determinant of $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ is:

A.	-1
B.	0
C.	1
D.	2

(51) The determinant of $A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is:

A. -1
B. 0
C. 1
D. 2

(52) The determinant of $A = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 2 & 5 \\ 3 & 3 & 6 \end{bmatrix}$ is:

A. -1
B. 0
C. 1
D. 2

For questions 5, 6, 7 and 8: Let $\mathbb{P}_2$ be the vector space of polynomials of degree 2 or less, so vectors have the form $f = a_0 + a_1 t + a_2 t^2$. Consider the inner product

$$\langle f, g \rangle := \int_0^1 f(t)g(t)dt.$$

For example $\langle 1, t \rangle = \int_0^1 1 \cdot t \; dt = \left(\frac{1}{2}t^2\right)|_0^1 = \frac{1}{2}$.

(53) If $f(t) = t$, then the length (or norm) $\|f\|$ is

A. $1/2$
B. $1/\sqrt{2}$
C. $1/3$
D. $1/\sqrt{3}$

(54) Let $f(t) = t$ and $g(t) = t^2 - \frac{3}{4}t$. What is the inner product $\langle f, g \rangle$?

A. $1/2$
B. $-3/4$
C. 0
D. $1/\sqrt{2}$

(55) If $V = \text{Span}(f) = \text{Span}(t)$, and $g(t) = t^2 - \frac{3}{4}t$, then the projection $\hat{g}$ of g onto V is

A. t^2
B. $-\frac{3}{4}t$
C. 0
D. $t^2 - \frac{3}{4}t$

(56) Still letting $V = \text{Span}(f) = \text{Span}(t)$ and $g(t) = t^2 - \frac{3}{4}t$, the projection $g^\perp$ of g onto $V^\perp$ is

$A.$ t^2

$B.$ $-\frac{3}{4}t$

$C.$ 0

$D.$ $t^2 - \frac{3}{4}t$

(57) Which of the following vectors is an eigenvector for the matrix

$$\begin{bmatrix} 2 & 1 \\ 0 & \frac{1}{2} \end{bmatrix}$$

with eigenvalue $\frac{1}{2}$?

A. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} -\frac{2}{3} \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix}$

(58) What is the determinant of

$$\begin{bmatrix} 2 & 2 & 4 \\ 6 & 5 & 0 \\ 0 & 1 & 3 \end{bmatrix}?$$

A. 18
B. -8
C. 22
D. -4
E. 30

(59) Is 1 an eigenvalue of the matrix

$$\begin{bmatrix} 3 & 2 & 4 \\ 6 & 6 & 0 \\ 0 & 1 & 4 \end{bmatrix}?$$

(Hint: Compare to the previous problem.)

A. Yes
B. No

(60) Suppose A is a matrix with real entries, such as $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Then the eigenvalues of A must be real.

A. True
B. False

(61) Let A be a 3×3 matrix so that $A\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Then A must have non-zero determinant.

A. True
B. False

(62) Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

Then A^{-1} is equal to A^T.

A. True
B. False

(63) Suppose A is a 2×2 matrix and it has a basis of eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$. Then A **must be** symmetric.

A. True
B. False

(64) Suppose A is a 2×2 matrix and it has a basis of eigenvectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Then A **must be** symmetric.

A. True
B. False

(65) Suppose A is a 3×3 matrix with eigenvalues 0, 2 and 7. Then

(a) A must be invertible
(b) A must be non-invertible
(c) Not enough information to tell

(66) Suppose A is a 3×3 matrix so that $\begin{bmatrix}1\\4\\2\end{bmatrix}$ is an eigenvector with eigenvalue 3 and $\begin{bmatrix}0\\1\\0\end{bmatrix}$ is an eigenvector with eigenvalue 2. What is $A\begin{bmatrix}1\\5\\3\end{bmatrix}$?

(a) $\begin{bmatrix}1\\5\\2\end{bmatrix}$
(b) $\begin{bmatrix}3\\14\\6\end{bmatrix}$
(c) Not enough information to tell.

(67) Which matrix has exactly two eigenspaces: Span $\left\{\begin{bmatrix}2\\1\\0\end{bmatrix}\right\}$ corresponding to $\lambda = 3$ and Span $\left\{\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$ corresponding to $\lambda = 2$?

(a) $\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$ (b) $\begin{bmatrix}5&-4&0\\1&1&0\\0&0&2\end{bmatrix}$ (c) $\begin{bmatrix}5&-4&0\\1&1&0\\0&0&4\end{bmatrix}$ (d) $\begin{bmatrix}3&0&0\\0&3&0\\0&0&2\end{bmatrix}$ (e) None of the above.

(68) Which of the following statements are true?
A. If M is a 4×4 matrix with eigenvalues 1, 2, 3 and 4, then M must have an eigenbasis.
B. If M is a 4×4 symmetric matrix with eigenvalues 1, 2, 3 and 3, then M must have an eigenbasis.

(a) Both A and B are true
(b) A is true but B is false
(c) B is true but A is false
(d) Neither A nor B is true

(69) Suppose A is a 5×5 **symmetric** matrix, and 1 is an eigenvalue with eigenspace Span $\left\{\begin{bmatrix}3\\0\\1\\2\\0\end{bmatrix}\right\}$. Suppose the **only** other eigenvalue of A is 2. What are the possible dimensions of the eigenspace of 2?

(a) 0 only

(b) 1 only
(c) 0, 1, 2, 3, or 4 only
(d) 1, 2, 3 or 4 only
(e) 4 only

(70) Suppose A is a 2×2 matrix,
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
and that $\mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is an eigenvector with eigenvalue 2. Is $\mathbf{v}$ also an eigenvector of the matrix $2A$, and if so, what is its eigenvalue?

(a) $\mathbf{v}$ is not necessarily an eigenvector of $2A$.
(b) $\mathbf{v}$ must be an eigenvector of $2A$, with eigenvalue $1/2$
(c) $\mathbf{v}$ must be an eigenvector of $2A$, with eigenvalue 2
(d) $\mathbf{v}$ must be an eigenvector of $2A$, with eigenvalue 4

(71) Suppose Q is an $m \times n$ matrix with orthonormal columns, and $m > n$. Which of the following statements must be true?
A. $QQ^T = I_m$, where I_m is the $m \times m$ identity matrix.
B. $Q^TQ = I_n$, where I_n is the $n \times n$ identity matrix.

(a) Both A and B are true
(b) A is true but B is false
(c) B is true but A is false
(d) Both A and B are false

If f and g are functions defined on $[0, 4\pi]$ we define their dot product to be
$$\langle f, g\rangle = \int_0^{4\pi} f(x)g(x)\ dx.$$
For numbers 13 and 14 below, consider the functions
$$f(x) = \begin{cases} 1 & x \in [0, \pi) \\ 0 & x \in [\pi, 4\pi] \end{cases}, \qquad g(x) = \sin x.$$
(72) What is the dot product $\langle f, g\rangle$?

(a) 2π
(b) π
(c) 2
(d) 1
(e) 0

(73) Note that $\langle g, g\rangle = 2\pi$. What is the $\sin x$ term of the Fourier series for f? In other words, the orthogonal projection of f onto g?

(a) $2\sin x$
(b) $\frac{1}{\pi}\sin x$
(c) $\frac{1}{\sqrt{2\pi}}\sin x$
(d) $\sqrt{\frac{2}{\pi}}\sin x$
(e) 0

(74) The determinant of the matrix $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is:

(a) 3
(b) -3
(c) 1
(d) -1
(e) 0

(75) If A is a square matrix and $\det(A) = 5$, then $\det(2A)$ must be

(a) 5
(b) 10
(c) 25
(d) 40
(e) Not enough information to tell.

(76) If A is a 3×3 matrix and $A \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ then $\det(A)$ must be

(a) 0
(b) 1
(c) -1
(d) 2
(e) Not enough information to tell.

(77) The eigenvalues of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ are:

(a) 1 and 0
(b) 1 and 6
(c) 2 and 3
(d) 3.5 and -3.5
(e) 0 and 7

(78) If $f : \mathbb{R}^2 \to \mathbb{R}$ is a twice-differentiable function, with a critical point at $\mathbf{0}$, and its Hessian matrix at this point is $H = \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$ then

(a) f must have a local minimum at 0
(b) f must have a local maximum at 0
(c) f cannot have a local minimum or maximum at 0
(d) Not enough information to tell

(79) True or False: If A and B are two invertible $n \times n$ matrices, then

$$(AB)^{-1} = B^{-1}A^{-1}.$$

(a) True
(b) False

(80) True or False: If A, B and C are three $n \times n$ matrices, then

$$(ABC)^2 = A^2B^2C^2.$$

(a) True
(b) False

(81) True or False: Every invertible $n \times n$ matrix A can be written as a product of elementary matrices:

$$A = E_1E_2 \cdots E_r$$

(a) True
(b) False

(82) True or False: If E is an elementary matrix, then E^{-1} is also an elementary matrix.

(a) True
(b) False

(83) True or False: If A is an $n \times n$ matrix and the columns of A are linearly independent, then A is invertible.

(a) True
(b) False

(84) True or False: If $\begin{bmatrix} a \\ b \end{bmatrix}$ and $\begin{bmatrix} c \\ d \end{bmatrix}$ are linearly independent, then $\begin{bmatrix} a \\ c \end{bmatrix}$ and $\begin{bmatrix} b \\ d \end{bmatrix}$ must be linearly independent.

(a) True
(b) False

(85) True or False: If A is an $m \times n$ matrix and an echelon form U has a bottom row consisting entirely of zeroes, then the columns of A must be linearly **dependent**.

(a) True
(b) False

(86) True or False: If A is an $m \times n$ matrix and an echelon form U has a bottom row consisting entirely of zeroes, then the columns of A do **not** span all of $\mathbb{R}^m$.

(a) True
(b) False

(87) True or False: If V is a vector space of dimension n, and $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are n different vectors that together span V, then they must also be linearly independent.

(a) True
(b) False

(88) True or False: If $\mathbf{v}$ and $\mathbf{w}$ are perpendicular vectors in $\mathbb{R}^2$, then the dot product $\mathbf{v} \cdot \mathbf{w}$ must be zero.

(a) True
(b) False

(89) True or False: If A is an $m \times n$ matrix, then the column space $\mathrm{Col}(A)$ and the left null space $\mathrm{Nul}(A^T)$ are orthogonal complements.

(a) True
(b) False

(90) True or False: If $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$, $\mathbf{w}$ is in the span of $\mathbf{v}$, and if $\hat{\mathbf{w}}$ is the projection of $\mathbf{w}$ onto $\mathbf{v}$, then

$$\mathrm{Span}\{\mathbf{v}, \mathbf{w}\} = \mathrm{Span}\{\mathbf{v}, \hat{\mathbf{w}}\}$$

(a) True
(b) False

(91) True or False: If $\mathbf{v}$ and $\mathbf{w}$ are vectors in $\mathbb{R}^n$, $\mathbf{w}$ is **not** in the span of $\mathbf{v}$, and if $\mathbf{w}^{\perp}$ is the projection of $\mathbf{w}$ onto the orthogonal complement of $\mathrm{Span}\{\mathbf{v}\}$, then

$$\mathrm{Span}\{\mathbf{v}, \mathbf{w}\} = \mathrm{Span}\{\mathbf{v}, \mathbf{w}^{\perp}\}$$

(a) True

(b) False

(92) True or False: For every square matrix A, both A and A^T have the same determinant.

(a) True
(b) False

(93) True or False: Rearranging the rows of A does not change its determinant.

(a) True
(b) False

(94) True or False: If A is a square matrix, then its null space $\text{Nul}(A)$ is also one of the eigenspaces of A.

(a) True
(b) False

(95) True or False: Every square matrix A has a diagonalization $A = PDP^{-1}$, where D is a diagonal matrix.

(a) True
(b) False

(96) True or False: If A is symmetric, then all its eigenvalues must be real.

(a) True
(b) False

(97) True or False: If f is a twice-differentiable function with a critical point at 0, and every entry in its Hessian matrix H is positive, then f must have a local minimum at 0.

(a) True
(b) False

(98) True or False: If A is a Markov matrix, then A must have $\lambda = 1$ as an eigenvalue.

(a) True
(b) False

(99) Let

$$V = \left\{ \begin{bmatrix} 2a+3b \\ 2c \\ 3a+c \end{bmatrix} \ : \ a,b,c \in \mathbb{R} \right\}$$

Which of the following is a basis for V?

(a) $\left\{ \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$

(b) Span $\left\{ \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$

(100) The matrix A can be put into Echelon form using the following row operations:

$$A = \begin{bmatrix} 1 & 0 & 3 \\ -3 & 4 & -7 \\ 2 & 4 & 5 \end{bmatrix} \xrightarrow{R2 \to R2+3R1} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 4 & 2 \\ 2 & 4 & 5 \end{bmatrix} \xrightarrow{R3 \to R3-2R1} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 4 & 2 \\ 0 & 4 & -1 \end{bmatrix}$$

$$\xrightarrow{R3 \to R3-R2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 4 & 2 \\ 0 & 0 & -3 \end{bmatrix} = U$$

What is the matrix L in the LU decomposition of A corresponding to the above U?

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -5 & -1 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$

(101) Suppose A is a 3×3 matrix with $A \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Is A invertible?

(a) Yes
(b) No
(c) Not enough information to tell

(102) Suppose A is a 3×3 matrix with $A \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$. Is A invertible?

(a) Yes
(b) No
(c) Not enough information to tell

(103) Select the inverse of $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$:

(a) $\begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$ (b) $\frac{1}{4}\begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 1 \\ 2 & -2 \end{bmatrix}$ (d) $\frac{1}{4}\begin{bmatrix} -3 & 1 \\ 2 & -2 \end{bmatrix}$

(104) Suppose A is a 3×3 matrix with columns $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{v}_3$:

$$A = \begin{bmatrix} | & | & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \\ | & | & | \end{bmatrix}$$

and $\text{Nul}(A) = \text{Span}\left\{\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}\right\}$. Are the columns of A linearly dependent, and if so, what is a nontrivial dependence relation between them?

(a) The columns of A are linearly independent
(b) The columns of A are linearly dependent and a dependence relation is

$$0\mathbf{v}_1 + 0\mathbf{v}_2 + 0\mathbf{v}_3 = \mathbf{0}$$

(c) The columns of A are linearly dependent and a dependence relation is

$$4\mathbf{v}_1 - 6\mathbf{v}_2 + 2\mathbf{v}_3 = \mathbf{0}$$

(d) The columns of A are linearly dependent, but neither of the above options is a nontrivial dependence relation

(105) Which of the following are subspaces of the indicated vector space:

A. If A is a 2×2 matrix, $\{\mathbf{x} \mid A\mathbf{x} = 3\mathbf{x}\}$, as a subset of $\mathbb{R}^2$
B. If A is a 2×2 matrix, $\{\mathbf{x} \mid Ax = \begin{bmatrix} 2 \\ 0 \end{bmatrix}\}$, as a subset of $\mathbb{R}^2$

(a) Only A is a subspace
(b) Only B is a subspace
(c) Both A and B are subspaces
(d) Neither A nor B are subspaces

(106) The matrix

$$A = \begin{bmatrix} 98 & -225 & -309 & 13 \\ 56 & -114 & -162 & 22 \\ -14 & 21 & 33 & -13 \\ 14 & -27 & -39 & 7 \end{bmatrix}$$

has Echelon form

$$U = \begin{bmatrix} 7 & 1 & -5 & 18 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Which of the following is a basis for the column space of A?

A. $\left\{ \begin{bmatrix} 98 \\ 56 \\ -14 \\ 14 \end{bmatrix}, \begin{bmatrix} -225 \\ -114 \\ 21 \\ -27 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 7 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 98 \\ 56 \\ -14 \\ 14 \end{bmatrix}, \begin{bmatrix} -225 \\ -114 \\ 21 \\ -27 \end{bmatrix}, \begin{bmatrix} -309 \\ -162 \\ 33 \\ -39 \end{bmatrix}, \begin{bmatrix} 13 \\ 22 \\ -13 \\ 7 \end{bmatrix} \right\}$

(a) A only
(b) B only
(c) A and B only
(d) A and C only

(107) If A is a 3×6 matrix of rank 2, then $\mathrm{Nul}(A)$ has dimension

(a) 0
(b) 1
(c) 2
(d) 3
(e) 4

(108) The matrix

$$A = \begin{bmatrix} 2 & 3 & 3 & 4 \\ 4 & 6 & 6 & 8 \\ 2 & 5 & 7 & 7 \end{bmatrix}$$

has Echelon form

$$U = \begin{bmatrix} 2 & 3 & 3 & 4 \\ 0 & 2 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Which of the following are a basis for $\mathrm{Col}(A^T)$?

A. $\left\{ \begin{bmatrix} 2 \\ 3 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 6 \\ 8 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 2 \\ 3 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 4 \\ 3 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 2 \\ 3 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 6 \\ 8 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 7 \\ 7 \end{bmatrix} \right\}$

(a) A only
(b) B only
(c) A and B only
(d) A and C only

(109) For every 5×5 matrix A, if

$$\dim \operatorname{Col}(A^T) = 3,$$

then the multiplicity of the eigenvalue $\lambda = 0$ of A

(a) must be 2.
(b) must be 3.
(c) can be either 0, 1, or 2.
(d) can be either 3, 4, or 5.
(e) can be either 2, 3, 4, or 5.

(110) Which of the following maps $T : \mathbb{R}^2 \to \mathbb{R}^2$ are linear?

A. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+1 \\ x+1 \end{bmatrix}$
B. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x-3y \\ x \end{bmatrix}$
C. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \\ y^2 \end{bmatrix}$
D. Rotation by an angle of α about the origin

(a) A, B, C, and D
(b) A, B, and D only
(c) B and D only
(d) A and B only
(e) None of the above

(111) Suppose $f : \mathbb{P}_1 \to \mathbb{P}_1$ is a linear map that has matrix

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$$

with respect to input and output bases $\{1, t\}$. What is $f(1 + 3t)$?

(a) $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$
(b) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$
(c) $3 + 5t$

(d) $-1+t$

(112) Let L be the linear transformation from $\mathbb{P}_2$ to $\mathbb{P}_2$ given by

$$L(p(t)) = p'(t) + 4p(t)$$

and let $\mathcal{B} = \{1, t, t^2\}$ be the standard basis for $\mathbb{P}_2$. Then the coordinate matrix $L_{\mathcal{B}\mathcal{B}}$ representing L with respect to input and output basis $\mathcal{B}$ is:

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 8 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 0 & 0 \\ 1 & 4 & 0 \\ 0 & 2 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 8 \\ 0 & 0 & 1 \end{bmatrix}$ (e) None of the above.

(113) Given $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 4 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$ what is the projection of $\mathbf{v}$ onto $\mathbf{w}$?

(a) $\frac{5}{4}\begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$ (b) $\frac{1}{4}\begin{bmatrix} -1 \\ 13 \\ 3 \\ 11 \end{bmatrix}$ (c) $\frac{5}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$ (d) $\frac{1}{5}\begin{bmatrix} 1 \\ 2 \\ 2 \\ 4 \end{bmatrix}$ (e) None of the above

(114) If $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is an orthogonal basis of $\mathbb{R}^4$ and $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$, then the coordinate matrix $P_{\mathcal{B}\mathcal{B}}$ representing the projection map onto W with input and output basis $\mathcal{B}$ is:

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (d) None of the above

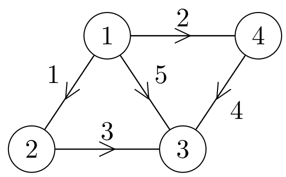
(115) Consider the orthonormal basis $\mathcal{B} = \left\{ \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \right\}$ of $\mathbb{R}^3$. What are the coordinates of the vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ in this basis?

(a) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ (c) $\begin{bmatrix} 1/2 \\ 1/3 \\ 1/6 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}$

(116) If V is a 13 dimensional subspace of $\mathbb{R}^{20}$, then the dimension of $V^{\perp}$ must be

(a) 0
(b) 7
(c) 13
(d) 20

(117) If A is the edge-node incidence matrix for the graph



then the dimension of $\mathrm{Nul}(A)$ is

(a) 0
(b) 1
(c) 2
(d) 4
(e) 5

(118) Consider the three row vectors $\mathbf{a}^T, \mathbf{b}^T, \mathbf{c}^T$, where $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbb{R}^3$. Let

$$A = \begin{bmatrix} \text{——} & \mathbf{a}^T & \text{——} \\ \text{——} & \mathbf{b}^T & \text{——} \\ \text{——} & \mathbf{c}^T & \text{——} \end{bmatrix}$$

be a matrix with determinant 3. What is the determinant of the matrix

$$B = \begin{bmatrix} \text{——} & 2\mathbf{a}^T & \text{——} \\ \text{——} & 2\mathbf{a}^T + 2\mathbf{c}^T & \text{——} \\ \text{——} & 2\mathbf{b}^T + 10\mathbf{c}^T & \text{——} \end{bmatrix}?$$

(a) -6
(b) 6
(c) -24
(d) 24

(119) If $\mathcal{A}$ and $\mathcal{B}$ are bases, and $I_{\mathcal{B}\mathcal{A}} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, then the equation $\begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ means:

(a) If $\mathbf{v}$ has $\mathcal{A}$-coordinates $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, then it has $\mathcal{B}$-coordinates $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$.
(b) If $\mathbf{v}$ has $\mathcal{B}$-coordinates $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$, then it has $\mathcal{A}$-coordinates $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$.
(c) If $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, then it has $\mathcal{B}$-coordinates $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$.

(d) If $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, then it has $\mathcal{A}$-coordinates $\begin{bmatrix} 5 \\ 5 \end{bmatrix}$.