Math 304 Quiz 8 — Section 5 — March 1, 2019

1. Which of the following transformations cannot be made via a sequence of elementary row operations?

   (a) $\begin{bmatrix} 2 & 7 \\ 1 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 7 \\ 3 & 8 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 1 \\ 2 & 7 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 & 2 \\ 7 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} -11 & 2 \\ 1 & 1 \end{bmatrix}$

   (d) All of the above transformations can be made via a sequence of elementary row operations.

2. If one abbreviates a system of m linear equations in n unknowns as $A\mathbf{x} = \mathbf{b}$, then

   (a) A is an $m \times n$ matrix and $b \in \mathbf{R}^n$

   (b) A is an $m \times n$ matrix and $b \in \mathbf{R}^m$

   (c) A is an $m \times n$ matrix and $b \in \mathbf{R}^n$ or $b \in \mathbf{R}^m$

   (d) A is an $n \times m$ matrix and $b \in \mathbf{R}^n$

3. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If R is the reduced row echelon form of the augmented matrix for the system $A\mathbf{x} = \mathbf{b}$, what are the solutions to that system?

   (a) $x_1 = 1, x_2 = 1$, and $x_3 = 2$

   (b) $x_1 = 1, x_2 = 1, x_3 = 2$, and $x_4 = 0$

   (c) $x_1 = -t, x_2 = -t, x_3 = -2t$, and $x_4 = t$

   (d) There are no solutions to this system

4. Let $R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$. If R is the reduced row echelon form of the coefficent matrix for the system $A\mathbf{x} = \mathbf{0}$, what are the solutions to that system?

   (a) $x_1 = 1, x_2 = 1$, and $x_3 = 2$

   (b) $x_1 = 1, x_2 = 1, x_3 = 2$, and $x_4 = 0$

   (c) $x_1 = -t, x_2 = -t, x_3 = -2t$, and $x_4 = t$

   (d) There are no solutions to this system

5. Let A, B, C be 3 matrices such that the product ABC is defined. What is $(ABC)^T$?

   (a) $(ABC)^T = A^T B^T C^T$

   (b) $(ABC)^T = B^T C^T A^T$

(c) $(ABC)^T = C^T A^T B^T$

(d) $(ABC)^T = C^T B^T A^T$

6. Which of the following matrices does not have an inverse?

(a) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 4 \\ 2 & 0 \end{bmatrix}$

(d) More than one of the above do not have inverses

(e) All have inverses

7. We find that for a square coefficient matrix A, the homogenous matrix equation $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, has only the trivial solution $X = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. This means that

(a) The matrix A has no inverse.

(b) The matrix A has an inverse.

(c) This tells us nothing about whether A has an inverse.

8. We know that $(5A)^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. What is the matrix A?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$

(d) $\begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$

(e) There is no matrix A which solves this equation.

9. If A is an invertible matrix, what else must be true?

(a) If $AB = C$ then $B = A^{-1}C$

(b) A^2 is invertible

(c) A^T is invertible

(d) The reduced row echelon form of A is I

(e) All of the above must be true

10. The reduced row-echelon form of the matrix

$$A = \begin{bmatrix} -2 & 2 & 3 & 1 & -3 & -2 \\ 0 & 2 & 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & -3 & -2 & 2 \end{bmatrix}$$

is given by

$$R = \begin{bmatrix} 1 & 0 & 0 & 10 & 2 & -7 \\ 0 & 1 & 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 1 & 7 & 0 & -5 \end{bmatrix}$$

Let F be the linear transformation $F\colon \mathbf{R}^6 \to \mathbf{R}^d$ given by $F(x) = Ax$. The number d is

(a) 6

(b) 2

(c) 5

(d) 4

(e) 3

11. Consider the system of linear equations:

$$2x_1 + 3x_2 + 4x_3 = 5$$
$$3x_1 + 2x_2 + x_3 = 0$$
$$x_1 + 2x_2 + 2x_3 = 2$$

Another way to express this system is:

(a) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix} x_2 + \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} x_3 = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} x_1 + \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} x_2 + \begin{bmatrix} 1 & 2 & 2 \end{bmatrix} x_3 = \begin{bmatrix} 5 & 0 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 2 \end{bmatrix}$

12. Suppose that A is 3×4. Then the number of solutions to the system $A\mathbf{x} = 0$ is

    (a) infinite

    (b) one

    (c) zero

    (d) indeterminable without more information

13. If one multiplies a row vector of length 5 by a column vector of length 5, in that order (row times column), one gets

    (a) A number

    (b) A row vector

    (c) A column vector

    (d) A matrix

    (e) Nothing; this operation cannot be defined in general

14. If one multiplies a column vector of length 5 by a row vector of length 5, in that order (column times row), one gets

    (a) A number

    (b) A row vector

    (c) A column vector

    (d) A matrix

    (e) Nothing; this operation cannot be defined in general