Math 304 Section 5 Quiz 9—March 6

1. Let $B = (v_1, v_2)$ be an ordered basis for $\mathbb{R}^2$. Which sentence below gives the meaning of the notation $\begin{pmatrix} a \\ b \end{pmatrix}_B$?

   (a) It is the selection of points based on the lengths of vectors in a matrix.

   (b) It gives the scalar values for the horizontal and vertical axes.

   (c) It means that a and b exist in the axis of B.

   (d) In the basis B, the numbers a and b lie in $\mathbb{R}^2$.

   (e) None of the above interpretations of this notation is correct.

2. Let A be a 2×2 matrix and consider the following properties that A might possess:

   I $\operatorname{Tr} A^T A \neq 0$

   II A^{-1} exists.

   Which of the following statements about property I and property II is most accurate?

   (a) I and II are equivalent.

   (b) I and II are both true for all 2×2 matrices.

   (c) I is true for all 2×2 matrices, but II is not.

   (d) I implies II, but not the other way around.

   (e) II implies I, but not the other way around.

3. Let $B = (v_1, \ldots, v_n)$ and $C = (w_1, \ldots, w_m)$ be ordered bases for $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively and let T be a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$. Then $_C T_B$, the matrix of T with respect to the input basis B and the output basis C, is best described as:

   (a) The matrix whose i-th column is the coordinates of v_i with respect to the basis C.

   (b) The matrix whose i-th row is the coordinates of v_i with respect to the basis C.

   (c) The matrix whose i-th row is the coordinates of $T(w_i)$ with respect to the basis B.

   (d) The matrix whose i-th column is the coordinates of w_i with respect to the basis B.

   (e) The matrix whose i-th column is the coordinates of $T(v_i)$ with respect to the basis C.

4. What are all the solutions to the system $x + 2y = 3$ and $2x + 4y = 5$?

   (a) $x = 0$ and $y = 1$

   (b) $0 = -1$

   (c) x can be arbitrary, and $y = \frac{5-2x}{4}$

   (d) x can be arbitrary, and $y = \frac{3-x}{2}$

   (e) There are no solutions.

5. Which of the following operations on an augmented matrix could change the solution set of a system?

   (a) Interchanging two rows

   (b) Multiplying one row by any constant

   (c) Adding one row to an other

   (d) Adding a multiple of one row to an other

   (e) None of the above

6. Complete the following phrase to make a true statement: "A homogeneous linear system of 2019 linear equations in 1776 unknowns..."

   (a) is always consistent

   (b) always has a unique solution

   (c) may be inconsistent

   (d) which is consistent always has a unique solution

   (e) which is consistent never has a unique solution

7. Consider a homogeneous linear system with n unknowns. Suppose the reduced row echelon form of its augmented matrix has $r \leq n$ nonzero rows. We can conclude that:

   (a) $x_1 = 0$, $x_2 = 0, \ldots, x_n = 0$ is a solution to the system

   (b) The system has $n - r$ free variables (parameters)

   (c) The system has infinitely many solutions

   (d) None of the above

   (e) More than one of the above

8. A vector subspace does *not* have to satisfy which of the following properties?

   (a) Associativity under vector addition

   (b) Existence of an additive identity

   (c) Commutativity under vector addition

   (d) A vector subspace must satisfy all of the above properties

   (e) A vector subspace need not satisfy any of the above properties