Math 304, Section 5 — Quiz 13 – April 1

Name:_____

1. A basis of a vector space V is a subset of V with two properties. What are the two properties?

2. If the columns of A are not linearly independent, how many solutions are there to the system $A\mathbf{x} = \mathbf{0}$?

(a) 0
(b) 1
(c) Infinitely many
(d) Not enough information given

3. Which of the following sets of vectors spans $\mathbb{R}^3$?

i.) $\begin{bmatrix}-2\\1\\3\end{bmatrix}, \begin{bmatrix}3\\5\\-1\end{bmatrix}$

ii.) $\begin{bmatrix}-2\\0\\4\end{bmatrix}, \begin{bmatrix}0\\1\\-1\end{bmatrix}, \begin{bmatrix}1\\2\\0\end{bmatrix}$

iii.) $\begin{bmatrix}2\\0\\0\end{bmatrix}, \begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}3\\0\\-2\end{bmatrix}, \begin{bmatrix}6\\2\\-2\end{bmatrix}$

iv.) $\begin{bmatrix}4\\6\\2\end{bmatrix}, \begin{bmatrix}3\\2\\1\end{bmatrix}, \begin{bmatrix}8\\12\\4\end{bmatrix}, \begin{bmatrix}6\\4\\2\end{bmatrix}$

(a) i, ii, iii, and iv
(b) ii, iii, and iv only
(c) ii and iii only
(d) i, ii and iii only
(e) iii and iv only
(f) ii only

4. Which of the following sets is/are linearly independent?

(a) $\left\{ \begin{bmatrix}1\\-1\\4\end{bmatrix}, \begin{bmatrix}2\\-2\\0\end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -12 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

5. Let $\mathbf{v} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, and $\sim 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ Which of the following are *linearly dependent*? Select all that apply

(a) $\{\mathbf{0}\}$

(b) $\{\mathbf{v}\}$

(c) $\{\mathbf{w}, \mathbf{0}\}$

(d) $\{\mathbf{v}, \mathbf{0}\}$

(e) $\{\mathbf{v}, \mathbf{w}\}$

(f) $\{\mathbf{v}, \mathbf{w}, \mathbf{0}\}$

6. Which of the following subsets $U \subset \mathbb{R}^n$ is a subspace?

(a) $U = \{x \in \mathbb{R}^n \mid x_1 = \cdots = x_n\}$

(b) $U = \{x \in \mathbb{R}^n \mid x_1^2 = x_2^2\}$

(c) $U = \{x \in \mathbb{R}^n \mid x_1 = 1\}$

7. Which of the following statements are true?

I. A set $\{u, v, w\}$ of vectors is linearly independent if only if for scalars a,b,$c \in \mathbb{R}$, $au + bv + cw = 0$ implies $a = b = c = 0$

II. A set $\{u, v, w\}$ of vectors is linearly independent if and only if for scalars a,b,$c \in \mathbb{R}$, $au + bv + cw = 0$ when $a = b = c = 0$

III. A set $\{u, v, w\}$ of vectors is linearly independent if and only if u is not a linear combination of v and w

IV. $\{(1, -1), (1, 1)\}$ spans $\mathbb{R}^2$

V. $\{(1, 0, 1), (1, 1, 1), (2, 1, 2)\}$ spans $\mathbb{R}^3$

(a) I & IV

(b) II & IV

(c) I & II

(d) III & V

(e) III & II

(f) I & V