Quiz 17

1. Which of the following statements about the real matrix shown below is FALSE?

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 2 & 3 & 4 & 5 \\ 0 & 0 & 3 & 4 & 5 \\ 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

(a) A is invertible

(b) If $\mathbf{x} \in \mathbf{R}^5$ and $A\mathbf{x} = \mathbf{x}$, then $\mathbf{x} = \mathbf{0}$

(c) The last row of A^2 is $\begin{bmatrix} 0 & 0 & 0 & 0 & 25 \end{bmatrix}$

(d) A can be transformed into the 5×5 identity matrix by a sequence of elementary row operations

(e) $\det(A) = 120$

2. Suppose the determinant of a 2×2 matrix A is equal to 3. What is the determinant of $5A$?

(a) 3

(b) 9

(c) 15

(d) 75

(e) Not enough information is given

3. What is the determinant of the matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 100 & 1 & 0 & 0 \\ 100 & 200 & 1 & 0 \\ 100 & 200 & 300 & 1 \end{bmatrix}$?

(a) 100

(b) 200

(c) 1

(d) 0

4. if A is a 3×5 matrix, then the determinant of A is

(a) A 3×5 matrix

(b) A 5×3 matrix

(c) A number (possibly non-zero)

(d) A subspace of $\mathbf{R}^3$

(e) A subspace of $\mathbf{R}^5$

(f) Zero

(g) Undefined

5. Which of the following is the larger of the eigenvalues of the matrix $\begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}$?

(a) 4

(b) 5

(c) 6

(d) 10

(e) 12

6. In order to be able to discuss the "eigenvalues" of a linear map $f : V \to W$ at all, f must be

    (a) Onto (surjective).

    (b) One-to-one (injective).

    (c) Endomorphic, i.e. we must have $W = V$.

    (d) An isomorphism, i.e. both one-to-one and onto.

7. Compute the product $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^4 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

    (a) $\begin{bmatrix} 27 \\ 27 \end{bmatrix}$

    (b) $\begin{bmatrix} 81 \\ 81 \end{bmatrix}$

    (c) $\begin{bmatrix} 243 \\ 243 \end{bmatrix}$

    (d) $\begin{bmatrix} 729 \\ 729 \end{bmatrix}$

    (e) None of the above

8. For any integer n, what will this product be? $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}^n \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

    (a) $3^n \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

    (b) $(-1)^n \begin{bmatrix} -5 \\ 5 \end{bmatrix}$

    (c) $(-5)^n \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

    (d) $5 \begin{bmatrix} (-1)^n \\ (-1)^n \end{bmatrix}$

    (e) None of the above

    (f) More than one of the above

9. Vector x is an eigenvector of matrix A. If $x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $Ax = \begin{bmatrix} 4 \\ 12 \end{bmatrix}$, then what is the associated eigenvalue?

    (a) 1

    (b) 3

    (c) 4

    (d) Not enough information is given

10. What does it mean if 0 is an eigenvalue of a matrix A?

    (a) The determinant of A is zero

    (b) The columns of A are linearly dependent

    (c) There are an infinite number of solutions to the system $Ax = 0$

    (d) All of the above

    (e) None of the above