Math 304 **Midterm 1** **Spring 2019**

No books, no notes, no calculators. You must show work, unless the question is a true/false, multiple choice, or fill-in-the-blank question.

Name: ______________________________ Section: ________________

Section Number	Instructor	Meeting Time
1	David Biddle	8:00
2	Daniel Rossi	8:00
3	David Biddle	9:40
4	Thomas Kilcoyne	11:20
5	Dikran Karagueuzian	11:20
6	Thomas Kilcoyne	1:10
7	Matt Evans	2:50
8	Joshua Carey	4:40

Page:	2	3	4	5	6	Total
Points:	14	18	22	10	10	74
Score:						

1. Fill in the blanks in the following definitions and statements of results from the textbook.

   (a) (4 points) A linear transformation L from one vector space to another has two fundamental properties:

      1. For all vectors u and v, ____________________ = ____________________.

      2. For all vectors w and all scalars c, ____________________ = ____________________.

      Hint: The properties above are listed in Chapter 1 with the heading "The key to the whole class ...".

   (b) (6 points) A matrix is said to be in "reduced row echelon form" if the following conditions are met:

      0. All zero rows are below all non-zero rows.

      1. In each non-zero row, the leftmost non-zero entry, called a pivot, is 1.

      2. The pivot of any given row is always ____

         ____.

      3. The pivot is the only ____

         ____.

   (c) (4 points) Suppose $B = (v_1, v_2, \ldots, v_n)$ is an ordered basis for a vector space V. The notation below defines a vector in V which is given by the equation:

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}_B = \underline{\hspace{8cm}}$$

(d) (3 points) The *Cauchy-Schwarz inequality* states that for any two vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, the absolute value of the dot product, $|\mathbf{u} \cdot \mathbf{v}|$ ______________________________________.

(e) (3 points) Theorem 7.5.1 states that an $n \times n$ matrix M is invertible if and only if the system of n equations in n unknowns $M\mathbf{x} = \mathbf{0}$ ____.

2. (12 points) Let V be the vector space of polynomials of degree less than or equal to 2. Let B be the ordered basis $(x^2, x, 1)$ for V. Let $L\colon V \to V$ be the linear transformation $\frac{\mathrm{d}}{\mathrm{dx}}$.

   Find ${}_BL_B$, that is, the matrix of L with respect to the basis B (used as both the input basis and output basis).

3. (12 points) Let S and T be linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$ defined by

$$S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -2 & -2 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \qquad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -3 & 0 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Find the matrix of the composition $T \circ S$ (with respect to the standard basis of $\mathbb{R}^2$), that is, the function that sends

$$\begin{bmatrix} x \\ y \end{bmatrix} \quad \text{to} \quad T\left(S\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)\right)$$

4. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{x}$ be vectors such that

$$\mathbf{u} \cdot \mathbf{v} = 8, \quad \mathbf{u} \cdot \mathbf{w} = -7, \quad \mathbf{v} \cdot \mathbf{w} = 6, \quad \text{and} \quad -2\mathbf{u} + 6\mathbf{v} = \mathbf{x}.$$

(a) (3 points) Find the dot product $\mathbf{v} \cdot \mathbf{u}$.

(b) (7 points) Find the dot product $\mathbf{x} \cdot \mathbf{w}$.

5. (10 points) Give a geometric description of the following system of equations:

$$\begin{aligned} 15x + 9y - 15z &= -6 \\ 25x + 15y - 25z &= -10 \\ -35x - 21y + 35z &= 14 \end{aligned}$$

Hints: this was a homework question. A "geometric description" is something like "these equations represent two lines in the plane, which intersect at the origin."

6. (10 points) Given the following LU factorization of the matrix M:

$$M = \begin{bmatrix} -2 & -3 & 1 \\ 6 & 5 & 1 \\ -6 & 3 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 3 & -3 & 1 \end{bmatrix} \begin{bmatrix} -2 & -3 & 1 \\ 0 & -4 & 4 \\ 0 & 0 & 1 \end{bmatrix} = LU$$

Use this factorization to solve the system of equations:

$$\begin{bmatrix} -2 & -3 & 1 \\ 6 & 5 & 1 \\ -6 & 3 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -30 \\ 29 \end{bmatrix}$$