Multiple Choice Questions

There is no penalty for guessing. Three points per question, so a total of 48 points for this section.

1. What is the complete relationship between homogeneous linear systems of equations, and the zero solution (all unknowns equal to zero)?

   (a) If a solution to a homogeneous linear system exists at all, then the zero solution will be a solution

   (b) The zero solution is always a solution to both homogeneous and inhomogeneous linear systems

   (c) The zero solution can be a solution to both homogeneous and inhomogeneous linear systems, but only if the equations are solvable

   (d) The zero solution is never a solution to inhomogeneous linear systems, and may or may not be a solution to homogeneous linear systems

   (e) The zero solution is always a solution to homogeneous linear systems, and never a solution to inhomogeneous linear systems

2. Let V be a vector space, and let W be a subset of V. What does it mean when we say that W is closed under scalar multiplication?

   (a) Whenever x is in W and c is a scalar, then cx is in V.

   (b) Whenever x is in V and c is a scalar, then cx is in V.

   (c) Whenever x is in V and c is a scalar, then cx is in W.

   (d) Whenever x is in W and c is a scalar, then cx is in W.

   (e) If cx is in W and c is a scalar, then x is in W.

3. Which of the following statements is not an axiom for vector spaces?

   (a) For all $x, y \in V$ we have $x + y = y + x$

   (b) For all $x, y, z \in V$, we have $(x + y) + z = x + (y + z)$

   (c) For all $x, y, z \in V$, we have $(xy)z = x(yz)$

   (d) All of the above are axioms for vector spaces.

4. What is the solution to the following system of equations?

$$2x + y = 3$$
$$3x - y = 7$$

   (a) $x = 4$ and $y = -5$

   (b) $x = 2$ and $y = -1$

   (c) $x = 2$ and $y = \frac{1}{2}$

   (d) There are an infinite number of solutions to this system.

   (e) There are no solutions to this system.

5. A system of 5 linear equations in 7 variables could not have exactly _____ solutions.

   (a) 0

   (b) 1

   (c) infinite

   (d) More than one of these is impossible.

   (e) All of these are possible numbers of solutions.

6. Which augmented matrix represents the following system of equations? (The augmented matrix is constructed with order of variables x, y.)

$$x + 2y = 3$$
$$4y + 5x = 6$$

   (a) $\left[\begin{array}{cc|c} 0 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right]$

   (b) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right]$

   (c) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 5 & 4 & 6 \end{array}\right]$

   (d) $\left[\begin{array}{cc|c} 0 & 2 & 3 \\ 5 & 4 & 6 \end{array}\right]$

7. What is the solution to the system of equations represented with this augmented matrix? (Assume that the variables are x, y, z, in that order.)

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

   (a) $x = 2$, $y = 3$, $z = 4$

   (b) $x = -1$, $y = 1$, $z = 1$

   (c) There are infinitely many solutions.

   (d) There is no solution.

   (e) We can't tell without having the system of equations.

8. If A is a 3×3 matrix such that $A \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $A \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, then the product $A \begin{bmatrix} 6 \\ 7 \\ 8 \end{bmatrix}$ is

   (a) $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

   (b) $\begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$

   (c) $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

   (d) $\begin{bmatrix} 9 \\ 10 \\ 11 \end{bmatrix}$

   (e) Not uniquely determined by the information given

9. Calculate the matrix product $\begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$

   (a) $\begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & -2 \\ 2 & 5 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & 0 \\ 6 & -2 \end{bmatrix}$

   (d) None of the above answers is correct.

   (e) This matrix multiplication is not defined.

10. When we put a matrix A into reduced row echelon form, we get the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$. This means that

    (a) The matrix A has no inverse.

    (b) The matrix we have found is the inverse of the matrix A.

    (c) Matrix A has an inverse, but this isn't it.

    (d) This tells us nothing about whether A has an inverse.

11. Find a matrix A such that $\left(2A^T + \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

    and give its first row

(a) $(2, -1)$

(b) $(0, 0)$

(c) $(-1/2, 1/2)$

(d) $(0, 1/2)$

(e) $(1/2, 0)$

12. Which matrix product is defined?

(a) $\begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 & 2 \\ 1 & 2 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

13. If the augmented matrix $[A|\mathbf{b}]$ of a system $A\mathbf{x} = \mathbf{b}$ is row equivalent to $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

Which of the following is true?

(a) The system is inconsistent.

(b) $\mathbf{x} = \begin{bmatrix} 5 \\ -2 - s \\ 1 \end{bmatrix}$ is a solution for any value of s.

(c) $\mathbf{x} = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$ is the unique solution of the system.

(d) $\mathbf{x} = \begin{bmatrix} 5s \\ -2s \\ s \end{bmatrix}$ is a solution for any value of s.

(e) $\mathbf{x} = \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$ is the unique solution to the system.

14. If C is a $n \times 4$ matrix and $D = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, then the second column of the matrix CD is

(a) The same as the second column of C

(b) The sum of the first and second columns of C

(c) The sum of the second and fourth columns of C

(d) The same as the third row of D

(e) The sum of the first and the third columns of C

15. What is the dot product of the vectors $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$?

(a) $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$

(b) 5

(c) 0

(d) The dot product of this pair of vectors is not defined.

16. What can we say about two vectors whose dot product is negative?

(a) The vectors are orthogonal

(b) The angle between the two vectors is less than $90°$

(c) The angle between the two vectors is greater than $90°$

(d) None of the above statements is correct.