Multiple Choice Questions

There is no penalty for guessing. Four points per question, so a total of 64 points for this section.

1. What is the determinant of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix}$?

   (a) 2
   (b) 4
   (c) 6
   (d) None of the above

   Solution: (c). You can compute this by cofactor expansion or row reduction.

2. What is the determinant of the matrix $\begin{bmatrix} \lambda & \lambda & \lambda \\ \lambda & \lambda & \lambda \\ \lambda & \lambda & \lambda \end{bmatrix}$?

   (a) 0
   (b) λ
   (c) λ^3
   (d) None of the above

   Solution: (a). There are two identical rows.

3. Suppose the determinant of a 2×2 matrix A is equal to 5. What is the determinant of $2A$?

   (a) 5
   (b) 10
   (c) 20
   (d) 25
   (e) There is insufficient information to answer the question.

   Solution: (c). Two rows get multiplied by 2, so the determinant is multiplied by 2 twice.

4. Suppose the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has an eigenvalue 1 with associated eigenvector $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. What is $A^{50}x$?

(a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(b) $\begin{bmatrix} a^{50} & b^{50} \\ c^{50} & d^{50} \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(d) $\begin{bmatrix} 2^{50} \\ 3^{50} \end{bmatrix}$

(e) There is insufficient information to answer the question

Solution: (c). The eigenvector-eigenvalue equation is $Ax = 1x$, so $A^2x = AAx = Ax = 1x = x$. Continuing in this manner we find $A^{50}x = x$.

5. Which of the following is an eigenvector of $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$?

(a) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

(b) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) None of the above

Solution: (c). Check the eigenvector-eigenvalue equation, rather than working with the characteristic polynomial.

6. $\begin{bmatrix} 4/3 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$. What is the associated eigenvalue?

(a) 4/3

(b) 5

(c) -2

(d) None of the above

Solution: (b). Check the eigenvector-eigenvalue equation, rather than working with the characteristic polynomial.

7. Choose the correct completion of the following statement: If A and B are square matrices such that $AB = I$, then zero is an eigenvalue of

   (a) A but not of B

   (b) B but not of A

   (c) Both A and B

   (d) Neither A nor B

Solution: (d). If $AB = I$, both A and B are invertible, so cannot have 0 as an eigenvalue, by the eigenvalue-eigenvector equation plus Theorem 7.5.1.

8. How many subspaces does $\mathbb{R}^2$ have?

   (a) two: 0 and $\mathbb{R}^2$

   (b) four: 0, $\mathbb{R} \times 0$, $0 \times \mathbb{R}$ (the "axes"), and $\mathbb{R}^2$ itself

   (c) infinitely many

   (d) None of the above answers is correct

Solution: (c). Any line through the origin is a subspace and there are infinitely many of these.

9. To determine whether a set of n vectors from $\mathbb{R}^n$ is independent, we can form a matrix A whose columns are the vectors in the set and then put that matrix in reduced row echelon form. If the vectors are linearly independent, what will we see in the reduced row echelon form?

   (a) A row of all zeros

   (b) A row that has all zeros except in the last position

   (c) A column of all zeros

   (d) An identity matrix

Solution: (d). Refer to Theorem 11.1.1. The vectors will be linearly independent iff $\det A \neq 0$ iff A is invertible iff $\text{RREF}(A) = I$.

10. Which of the following sets of vectors forms a basis for $\mathbb{R}^3$? (Hint: at least one of these sets is a basis!)

i.) $\left\{ \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} \right\}$

ii.) $\left\{ \begin{bmatrix} -2 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

iii.) $\left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ -2 \end{bmatrix} \right\}$

iv.) $\left\{ \begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 8 \\ 12 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix} \right\}$

(a) ii, iii, and iv only

(b) ii and iii only

(c) i, ii, and iii only

(d) iii and iv only

(e) ii only

Solution: (e). A basis of $\mathbb{R}^3$ has 3 elements, so only (ii) is a possibility. We can check whether this set forms a basis by computing the determinant of the matrix assembled from these vectors.

11. Which of the following sets is linearly independent? (Hint: only one set is linearly independent!)

(a) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \right\}$

(b) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

(c) $\left\{ \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ -12 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

(e) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

Solution: (a). A set of two vectors is linearly independent iff one is not a multiple of another.

12. Let V be a vector space, and let S be a subset of V. What does it mean when we say that S spans V?

(a) The elements of S are all distinct from each other

(b) Every vector in V has exactly one representation as a linear combination of vectors in S

(c) S has at least as many elements as the dimension V

(d) S is a basis for V

(e) Every vector in V can be expressed as a linear combination of vectors in S

Solution: (e). This is the definition of "span".

13. Let V be a five-dimensional vector space, and let S be a subset of V consisting of three vectors. Then S

(a) May or may not be linearly independent, and may or may not span V

(b) Must be linearly dependent, but may or may not span$V$

(c) Must be linearly independent, but cannot span V

(d) Can span V, but only if it is linearly independent, and vice versa

(e) Cannot span V, but can be linearly independent or dependent

Solution: (e). A spanning set is at least as large as a linearly independent set, and linearly independent sets include bases, which must have 5 elements. Thus S cannot span. S could be linearly dependent; for example it could include the zero vector.

14. Let V be a three-dimensional vector space, and let S be a subset of V consisting of five vectors. Then S

(a) Can span V,but only if it is linearly independent, and vice versa

(b) Must be linearly independent, but cannot span V

(c) Must be linearly dependent, and must span V

(d) Must be linearly independent, but may or may not span V

(e) Must be linearly dependent, but may or may not span V

Solution: (e). A linearly independent set can only be as large as a spanning set. Spanning sets include bases, which contain 3 elements. Thus S must be linearly dependent. S might not span, for example it could consist of several different multiples of one vector.

15. Let A and B be the 2×2 matrices

$$A = \begin{bmatrix} 3 & 1 \\ 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$$

Which of the following statements is most accurate?

(a) Neither A nor B is diagonalizable.

(b) Both A and B are diagonalizable.

(c) A is diagonalizable but B is not.

(d) B is diagonalizable but A is not.

Solution: (c). A has two distinct eigenvalues and corresponding eigenvectors, which allow us to construct a basis consisting of eigenvectors. Thus A is diagonalizable. B has only one eigenvalue, and the corresponding eigenspace is one-dimensional, so it impossible to construct a basis consisting of eigenvectors. (This type of example, a "Jordan Block", was treated at the end of Chapter 13.) Thus B is not diagonalizable.

16. Let A and B be the 2×2 matrices

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}$$

Which of the following statements is most accurate?

(a) Neither A nor B is diagonalizable.

(b) Both A and B are diagonalizable.

(c) A is diagonalizable but B is not.

(d) B is diagonalizable but A is not.

Solution: (d). A is a 90-degree rotation in the plane and has characteristic polynomial λ^2+1, so there are no eigenvalues or eigenvectors. Therefore A is not diagonalizable. B is diagonalizable, for any number of reasons, one of which is that it is symmetric and all symmetric matrices are diagonalizable.