Math 447 **Test 1** **Fall 2015**

No books, no notes, only SOA-approved calculators. You must show work, unless the question is a true/false or fill-in-the-blank question. Numerical answers should be given to 3 places, e.g. 97.9%.

Name: ________________________________ Section: ____________________

Question	Points	Score
1	19	
2	14	
3	11	
4	11	
5	9	
6	7	
7	10	
8	13	
9	12	
10	13	
11	13	
Total:	132	

1. On today's episode of the "Monty Hall" game show, you are shown seven cards, all face-down, of which one is the $Q\heartsuit$, one is the $Q\diamondsuit$, and five are black spot cards. The objective is to select the one of the queens, which wins \$100.

   The host lets you select a face-down card, and then shows you, from the other six face-down cards, one black spot card, which he can always do, because he knows where all the cards are. He then offers you the opportunity to switch your choice to one of the five remaining face-down cards.

   We will suppose in what follows that you decide to switch your choice.

   Let G denote the event that your initial selection was one of the queens. Let W denote the event that your new selection (after switching) is one of the queens.

   (a) (3 points) Find $P(\bar{G})$, $P(W \mid \bar{G})$, and $P(W \mid G)$.

   (b) (3 points) Write an equation for $P(W)$ using the "Law of Total Probability".

   (c) (3 points) Find $P(W)$.

   (d) (10 points) Suppose that after you are shown a black spot card, but before you switch your choice, the host offers to show you another black spot card in return for an immediate payment. How much, if anything, should you be willing to pay to see another black spot card?

2. A population of voters contains 40% Republicans and 60% Democrats. It is reported that 30% of the Republicans and 70% of the Democrats favor an election issue. A person chosen at random from this

population is found to favor the issue in question. Find the conditional probability that this person is a Democrat.

Solve this problem according to the following scheme. Make sure to give an appropriate answer for each part.

(a) (3 points) Establish notation: name the relevant events.

(b) (2 points) In the notation of part (a), what information does the problem give you?

(c) (2 points) In the notation of part (a), what probability does the problem ask you to compute?

(d) (4 points) Write down the equation that lets you compute this answer, again in the notation of part (a).

(e) (3 points) Solve the problem, giving your answer as a percentage.

3. Let Y be a random variable with mean 11 and variance 9.

   (a) (2 points) Fill in the blank in the following statement of Tchebysheff's Theorem: If Y is a random variable with $E[Y] = \mu$ and $V[Y] = \sigma^2$, then

   $$P(|Y - \mu| < k\sigma) \geq \underline{\hspace{3cm}}$$

   (b) (2 points) What are μ and σ for the Y of this problem? Give numerical answers.

   (c) (3 points) For the Y of this problem, what inequality does Tchebysheff's Theorem give you for the probability $P(6 < Y < 16)$?

   (d) (4 points) For what value of C does Tchebysheff's Theorem guarantee that $P(|Y - 11| \geq C) \leq 0.09$?

4. Let X be a geometric random variable with parameter p. Write q for $1-p$. All your answers below should be given in terms of p and q.

   (a) (2 points) Give a formula for $p(k)$, where k is a positive integer.

   (b) (2 points) Find $P(X = 1)$ and $P(X = 3)$.

(c) (3 points) Write down an infinite series for the probability that X is odd.

(d) (4 points) Sum the series of the previous part to obtain a closed-form expression for the probability that X is odd.

5. Many utility companies promote energy conservation by offering discount rates to consumers who keep their energy usage below certain established subsidy standards. A recent EPA report notes that 70% of the island residents of Puerto Rico have reduced their electricity usage sufficiently to qualify for discounted rates. If five residential subscribers are randomly selected from San Juan, Puerto Rico, find the probability of each of the following events:

   (a) (3 points) All five qualify for the favorable rates.

   (b) (6 points) At least four qualify for the favorable rates.

6. The number of knots in a particular type of wood has a Poisson distribution with an average of 1.5 knots in 10 cubic feet of the wood.
   (a) (3 points) Find the probability that a 10-cubic-foot block of the wood has no knots.

   (b) (4 points) Find the probability that a 10-cubic-foot block of the wood has at most 1 knot.

7. Fill in the blanks as appropriate in the following statements.
   (a) (3 points) The definition of the moment generating function $m_Y(t)$ of a random variable Y is
   $$m_Y(t) = E[\underline{\hspace{4cm}}]$$
   (b) (4 points) The moment generating function of a random variable Y gets its name from the fact that the k-th derivative $m_Y^{(k)}(t)$ has the property that
   $$m_Y^{(k)}(\underline{\hspace{1.5cm}}) = E[\underline{\hspace{4cm}}]$$
   (c) (3 points) The negative binomial random variable with parameters r and p is the sum of ____________ independent __________________ random variables with parameter(s) __________________.

8. Two dice are tossed. (You may assume these are standard 6-sided dice, independent of one another, and not loaded.)
   (a) (2 points) What is the probability that the total shown is 10?

   (b) (2 points) What is the probability that the total shown is 8?

(c) (9 points) If the dice are tossed repeatedly, what is the probability that we obtain a total of 8 before obtaining a total of 7?

9. If two events, A and B, are such that $P(A) = 0.2$, $P(B) = 0.3$, and $P(A \cup B) = 0.4$, find the following quantities:

   (a) (3 points) $P(A \cap B)$

   (b) (3 points) $P(\bar{A} \cap \bar{B})$

   (c) (3 points) $P(\bar{A} \cup \bar{B})$

   (d) (3 points) $P(\bar{A} \mid B)$

10. A jury of 6 persons was selected from a group of 20 potential jurors, of whom 8 were African American and 12 were white. Assume the members of the jury are randomly selected from the 20 potential jurors. Let Y be the number of jurors selected who are African American.

   (a) (2 points) What is the distribution of Y? (Hint: we only learned about 6 types of random variables!)

   (b) (2 points) What is (are) the relevant parameter(s)?

   (c) (5 points) Find the probability that at most one of the jurors selected is African American.

   (d) (2 points) What is the mean of Y?

   (e) (2 points) What is the variance of Y?

11. We observe a sequence of independent identical trials with two possible outcomes on each trial, S (success) and F (failure), and with $P(S) = p$. Let Y be the number of the trial on which we observe the fifth success.

   (a) (2 points) What is the distribution of Y? (Hint: we only learned about 6 types of random variables!)

   (b) (2 points) What is (are) the relevant parameter(s)?

(c) (9 points) What value of p maximizes $P(Y = 11)$?