Math 447 **Test 2** **Fall 2019**

No books, no notes, only SOA-approved calculators. You must show work, unless the question is a true/false or fill-in-the-blank question, or you are told elsewhere in the statement of the problem that no work is necessary. You may give numerical answers either as reduced fractions, or as decimals accurate to 3 significant figures unless otherwise instructed.

Name: ______________________________ Section: ____________________

Section Number	Instructor	Meeting Time
1	Dai	8:00
2	Karagueuzian	11:20
3	Hanson	8:00

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	16	19	12	15	9	10	12	10	113
Score:										

1. (10 points) (Bertrand Box Puzzle) We have five identical-looking boxes, each of which contains two coins. Of the five boxes,

   - two boxes contain two silver coins each,
   - two boxes contain two gold coins each,
   - one box contains a silver coin and a gold coin.

   A box is selected at random and a coin drawn from the box. This coin turns out to be gold. What is the probability that the other coin in the selected box is also gold?

2. Two methods, A and B, are available for teaching a certain industrial skill. The failure rate is 20% for A and 10% for B. However, B is more expensive and hence is used only 30% of the time. (A is used the other 70%.)

   (a) (8 points) A worker was taught the skill by one of the methods but failed to learn it correctly. What is the probability that she was taught by method A?

   (b) (8 points) What is the probability that a worker, chosen at random from those who were taught the skill, failed to learn the skill?

3. (For this question you may wish to refer to the table of the normal distribution attached to the back of your test.) We take as our model of the grade point averages (GPAs) of students at a certain college a normal distribution with mean 2.4 and standard deviation 0.8. The college has exactly 10,000 students.

   (a) (3 points) If students possessing a GPA less than 1.9 are placed on probation, what percentage of the students will be on probation?

   (b) (5 points) If the college administration wishes to put students with the lowest 4% of GPAs on probation, what should the cutoff GPA for probation be?

   (c) (5 points) How many students, *according to the model,* have GPAs which are *less than zero?*

   (d) (3 points) In the real world, how many students will actually have a GPA which is *less than zero?*

   (e) (3 points) Should your answers to (c) and (d) be the same? Why or why not?

4. Suppose that Y_1 and Y_2 are discrete random variables whose joint probability function $p(y_1, y_2)$ is given by the following table:

	y_1	
y_2	0	1
0	2/5	1/15
1	2/15	1/15
2	4/15	1/15

(a) (4 points) Find the marginal probability function $p(y_1)$ of Y_1.

(b) (6 points) Find the conditional probability function $p(y_2 \mid y_1)$ of Y_2 given Y_1. Put your answer in a table, similar to the table above.

(c) (2 points) Are Y_1 and Y_2 independent? (Yes or no; no work required.)

5. Suppose Y_1 and Y_2 are continuous random variables with joint density function given by

$$f(y_1, y_2) = \begin{cases} e^{-(y_1+y_2)}, & \text{for } y_1 > 0 \text{ and } y_2 > 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) (2 points) Are Y_1 and Y_2 independent? (Yes or no; no work required.)

(b) (3 points) The marginal distribution of Y_1 is one of the continuous distributions studied in Chapter 4 and listed in the table attached to the end of this test. What is the name of this distribution and what are the value(s) of the defining parameters?

(c) (4 points) Find $E[Y_1 - Y_2]$. Use the back of the previous page if necessary.

(d) (6 points) Find $V[Y_1 - Y_2]$. Use the back of the previous page if necessary.

6. Suppose Y is a continuous random variable with density function $f(y)$ and distribution function $F(y)$.

   (a) (3 points) Express $P(2 < Y < 5)$ as an integral.

   (b) (3 points) Express $P(1 < Y < 4)$ using the distribution function.

   (c) (3 points) Find the derivative $\frac{d}{dy}F(y)$

7. Let Y_1 and Y_2 be two uncorrelated random variables with $V[Y_1] = a$, $V[Y_2] = b$. Define two new random variables $U_1 = Y_1 + Y_2$ and $U_2 = Y_1 - Y_2$.

   (a) (3 points) Find $\text{Cov}(U_1, U_2)$ in terms of a and b.

   (b) (3 points) Find the correlation coefficient of U_1 and U_2 in terms of a and b.

   (c) (4 points) For what values of a and b is this correlation coefficient zero?

8. Suppose Y is a continuous random variable with probability density function given by

$$f(y) = \begin{cases} 4y^2 e^{-2y}, & y > 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) (3 points) What kind of random variable is Y and what are the values of the defining parameters?

(b) (4 points) Find $E[Y]$ and $V[Y]$.

(c) (5 points) Find $E[\sqrt{Y}]$.

9. (10 points) I work in midtown and have two favorite restaurants, one uptown and one downtown. I leave work every day at a random time uniformly distributed between 5 and 6 pm. I go to the subway and take the uptown or downtown train, whichever arrives first, to one of the restaurants.

   The uptown and downtown trains arrive at the station with exactly the same frequency. But over a long period of time I find that I go to the uptown restaurant on about 90% of days, and the downtown restaurant on about 10% of days.

   How is this possible? (Hint: a correct answer will involve writing down a timetable for the trains.)