Homework #2

Math 455, Spring 2025

Due Monday, February 3

This assignment is a review of the t- and F-distributions, based on the problems in Wackerly, Section 7.2. You may refer to this textbook as necessary in your solutions.

Problem 1: Adapted from Wackerly 7.31.

(a) What is the definition of $F_{0.01}$? Give a specific reference in the textbook. (Hint: look at the appendix on the F-distribution.)

(b) Find $F_{0.01}$ for F-distributed random variables, all with 4 numerator degrees of freedom, but with denominator degrees of freedom of 10, 15, 30, 60, 120, and ∞. (You can use R rather than the tables in the back of Wackerly. You may need to think about how to communicate ∞ to R.) Display your answer in the form of a table.

(c) Refer to part (b). What do you observe about the values of $F_{0.01}$ as the number of denominator degrees of freedom increases?

(d) What is the definition of $\chi^2_{0.01}$? Give a specific reference in the textbook.

(e) What is $\chi^2_{0.01}$? for a χ^2-distributed random variable with 4 df?

(f) Divide the value of $\chi^2_{0.01}$ (4 df) from part (e) by the value of $F_{0.01}$ (numerator df $= 4$; denominator df $= \infty$). Explain why the value that you obtained is a reasonable value for the ratio. [Hint: Consider the definition of an F-distributed random variable given in Definition 7.3.]

Problem 2: Adapted from Wackerly 7.32.

(a) What is the definition of $t_{0.05}$? Give a specific reference in the textbook. (Hint: look at the appendix on the t-distribution.)

(b) Find $t_{0.05}$ for a t-distributed random variable T with 5 df.

(c) Refer to part (b). What is $P(T^2 > (t_{0.05})^2)$?

(d) Find $F_{0.10}$ for an F-distributed random variable with 1 numerator degree of freedom and 5 denominator degrees of freedom.

(e) Compare the value of $F_{0.10}$ found in part (d) with the value of $(t_{0.05})^2$ from parts (b) and (c).

(f) In Exercise 7.33, you will show that if T has a t distribution with ν df, then $U = T^2$ has an F distribution with 1 numerator degree of freedom and ν denominator degrees of freedom. How does this explain the relationship between the values of $F_{0.10}$ (1 num. df, 5 denom df) and $(t_{0.05})^2$ that you observed in part (e)?

Problem 3: Wackerly 7.33.

Problem 4: Adapted from Wackerly 7.35. You may quote results from 7.34 without proof. Suppose in the following parts that F is an F-distributed random variable with 50 numerator dof and 70 denominator dof.

(a) Find $E[F]$.

(b) Find $V[F]$.

(c) Use Tchebysheff's Inequality to find a bound on $P(F > 3)$.

(d) Use R to find the value of $P(F > 3)$.

Problem 5: Do problem 7.36 in Wackerly. Adapting the work of Example 7.7 and quoting the definition of the F-distribution may be helpful.

Problem 6: Do problem 7.37 in Wackerly. In answering the "why" questions you should quote specific results in the textbook.

Problem 7: Do problem 7.38 in Wackerly. In answering the "why" questions you should quote specific results in the textbook.

Problem 8: Do problem 7.39 in Wackerly. In answering the "provide reasons" questions you should quote specific results in the textbook.