Math 455 Class 16 February 26

Mistake in practice midterm

$$W = \sum_{i=1}^{5} (x_i - \bar{x}_i)$$

should be

$$W = \sum_{i=1}^{5} (x_i - \bar{x})^2$$

The midterm (of necessity) does NOT cover all the material : insufficient time.

Things that MIGHT be on the real midterm that are NOT on the practice midterm include:

- relationship between M.L.E and the least-squares
- Hill Criteria
- Model Matrix / Design Matrix.

If we write $Y_i = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$

We have a table of data:

Y	1	X_1	X_2
Y_1	1	$X_{1,1}$	$X_{2,1}$
Y_2	1	$X_{1,2}$	$X_{2,2}$
Y_3	1	$X_{1,3}$	$X_{2,3}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

This part is the "design matrix" or "model matrix"

R command : model.matrix.

If our model specified "no intercept" or $\beta_0 = 0$, model.matrix would return

$$\begin{bmatrix} X_{1,1} & X_{2,1} \\ X_{1,2} & X_{2,2} \\ X_{1,3} & X_{2,3} \\ \vdots & \vdots \end{bmatrix}$$

instead of the above (with col of 1s)

Hill Criteria:

One way to get multiple-choices Qs about the Hill Criteria is to ask a LLM.

If you do this: warning: the names of the Hill Criteria in the text are not universal. e.g. "Coherence" instead of "Plausibility".

The Hill Criteria are NOT an exact science: the connection may be causal even if some of the Hill Criteria are not satisfied.

It's also possible to have all the criteria satisfied and <u>not</u> have a causal connection (maybe you overlooked an important covariate?)

Another idea to prepare for midterm: paste questions from midterm in to LLM and tell it: give me an analogous question.

No further homework until after midterm 1.

The "Hat Matrix" gets its name because it "puts a hat" on Y.

$$\hat{Y} = H\cancel{X}Y \quad \text{where} \quad H = X(X^TX)^{-1}X^T$$

$\hat{Y}$ is called the "fitted values" vector.

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i \quad \text{in the 1-variable case.}$$

$\hat{Y}_i$ is the point <u>on</u> the regression line: it is our "point estimate" of Y given the particular X-value

Things to know about the hat matrix:

it is the projection matrix on the col space of X.

So $H^2 = H$. (This is also directly computable.)

$$H^2 = X(X^TX)^{-1}\underbrace{X^TX(X^TX)^{-1}}_{=I}X^T$$

$$= X(X^TX)^{-1}X^T = H.$$

and H is symmetric $H^T = H$.

Using your linear algebra (304)

$$H^T = \left(X(X^TX)^{-1}X^T\right)^T = (X^T)^T\left((X^TX)^{-1}\right)^T X^T$$

$$= X\left((X^TX)^T\right)^{-1}X^T$$

$$= X\left(X^T(X^T)^T\right)^{-1}X^T$$

$$= H.$$

Note that the residuals e_i are defined by

$$Y_i = \hat{\beta}_0 + \hat{\beta}_1 X_i + e_i \qquad \text{(1-var. case)}$$

or: $Y_i - \hat{Y}_i = e_i$

actual value of Y

point on regression line at same X-val.

So the vector of residuals e is:

$$e = (I-H)Y$$

Connection between $\hat{Y}$ and R^2 (Chap. 2) (p. 23)

$$R^2 = \mathrm{Cor}(\hat{Y}, Y)^2$$