Math 461 - September 19, 2022 - Quiz 9

Name:_____

1. What does it mean to say that a topological space X is "first countable"? Give the definition.

2. Give an example of a topological space X which is *not* first countable.

3. Let X, Y be topological spaces and let $f\colon X \to Y$ be a continuous function. Suppose $K \subset Y$ is closed. Show that $f^{-1}(K)$ is closed.

4. Suppose, with notation as in 3 above, that $\langle x_n \rangle_{n=1}^{\infty}$ is a sequence in X which converges to $x \in X$. Does $\langle f(x_n) \rangle_{n=1}^{\infty}$ converge in Y, and if so, to what? Give a proof or a counterexample.