Quiz: a) Define what it means that $f: X \to Y$ is a quotient map.

b) Let A be a subset of a topological space X. Define X/A. Prove that if X/A is Hausdorf then A is a closed subset of X.

c) Define the product topology on $X \times Y$, where X, Y are topological spaces.

Solution: a) We say that $f: X \to Y$ is a quotient map if the topology on Y is iduced by f, i.e. if $U \subseteq Y$ is open if and only if $f^{-1}(U)$ is open in X.

b) X/A is by definition X/R where R is the following relation on X: xRy iff either $x=y$ or both x,y are in A. This means that all elements in A form an equivalence class of R, hence they all are mapped to the same point $[A]$ by the quotient map $q: X \to X/A$. Since X/A is Hausdorff, the point $[A]$ is closed in X/A hence $q^{-1}(q\{[A]\}) = A$ is closed in X, as q is a continuous map

c) The collection of all subsets of $X \times Y$ of the form $U \times V$, where $U \subseteq X$ is open and $V \subseteq Y$ is open, is a basis of a topology on $X \times Y$ called the product topology.