MATH 488A: SAMPLE TEST

1. Test 2: Sample

Problem 1. Show that it is not possible to cover a 6×6 board with 1×2 dominos so that each of the 10 lines of length 6 that form the board (but do not lie along its border) bisects at least one domino. But show that we can cover a 5×6 board with 1×2 dominos so that each of the 9 lines of length 5 or 6 that form the board (but do not lie along its border) bisects at least one domino.

Problem 2. (25 points) There is a chess tournament with $2n$ players ($n > 1$). There is at most one match between each pair of players. If it is not possible to find three players who all play each other, show that there are at most n^2 matches. Conversely, show that if there are at most n^2 matches, then it is possible to arrange them so that we cannot find three players who all play each other.

Problem 3. (25 points) There are 2000 points on a circle and each point is given a number which is equal to the average of the numbers of its two nearest neighbors. Show that all the numbers must be equal.

Problem 4. (25 points) Let $a_1, a_2, \dots a_n$ be positive integers all of whose prime divisors are ≤ 13.

(a) (10 points) Show that if $n \geq 65$ then there exist two of these integers whose product is a perfect square.

(b) (15 points) Show that if $n \geq 193$ then there exist four of these integers whose product is a perfect fourth power.

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