Math ~~#~~ 525 Class 40 December 6

Quiz soln's next page.

~~Q: Is it possible that x^2~~

Course Evaluations Today

Math 525 — Quiz 19 – December 6

Name: Solutions

All these questions consider a matrix $B \in M_{3\times 3}(\mathbb{Q})$ with $B^6 = I$, as in Example (5) of p. 487.

1. The invariant factors of B are defined by considering a $\mathbb{Q}[x]$-module V and its invariant factor decomposition. What is V and how is that action of $\mathbb{Q}[x]$ on V defined?

$V = \mathbb{Q}^3$ if $f \in \mathbb{Q}[x]$ define $f \cdot v = f(B)v$

in $M_{3\times3}(\mathbb{Q})$ in $\mathbb{Q}^3$

2. The text states that $B^6 = I$, so the minimal polynomial of B divides $x^6 - 1$ in $\mathbb{Q}[x]$. Why does this follow?

Let m_B for the minimal polyn. Then we can divide $x^6 - 1$ by m_B in $\mathbb{Q}[x]$, so $x^6 - 1 = q(x) m_B(x) + r(x)$ where $\deg(r) < \deg(m_B)$. Now eval @ B, $r(B) = 0$ in $M_{3\times3}(\mathbb{Q})$. Since $\deg(r) < \deg(m_B)$ $r = 0$ in $\mathbb{Q}[x]$.

3. The factorization of $x^6 - 1$ in $\mathbb{Q}[x]$ is

$$x^6 - 1 = (x-1)(x+1)(x^2+x+1)(x^2-x+1)$$

Is it possible that the minimal polynomial of B is $x^2 + x + 1$? Why or why not?

No. Every invariant factor divides m_B, and the product of the invariant factors is the characteristic polyn, which has degree 3. Thus if m_B is irred. of degree 2 this is impossible.

4. If the minimal polynomial of B is $(x+1)(x-1)$, what lists of invariant factors are possible, and why?

$(x-1)$, $(x+1)(x-1)$

$(x+1)$, $(x+1)(x-1)$