

Math 525

Class 25

October 27

Quiz solutions next page

New homework is posted. Due Wed. Nov 1

Next quiz 10.4. Oct 30 Mon.

Notes on getting a course:

Suppose you want to have a course on

"Quantum Computing": go to Jia Zhao

with 4 friends and say "we will register".

Note on HW:

Q: What is the structure of the group $\left(\frac{\mathbb{F}_3[x]}{(x^2)} \right)^\times$

A: $\mathbb{F}_3[x]/(x^2)$ is a comm. ring.

Every elt. can be expressed as $a + bx + I$.

Where $I = (x^2)$.

Name: Solutions

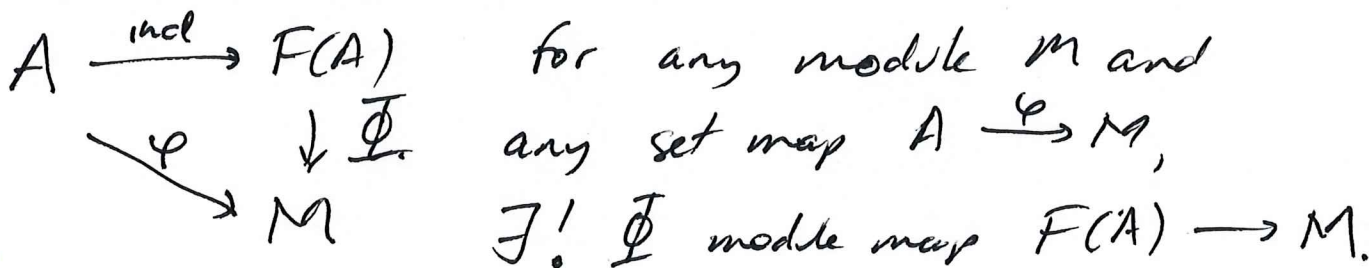
1. The inclusion of the rational numbers into the real numbers makes $\mathbb{R}$ a module over $\mathbb{Q}$. Is $\mathbb{R}$ a free $\mathbb{Q}$ -module? Why or why not?

Yes. $\mathbb{Q}$ is a field, so a $\mathbb{Q}$ -module is a vector space over $\mathbb{Q}$, and all vector spaces are free because AC gives a basis.

2. If V is a finite dimensional vector space over $\mathbb{R}$ and $T: V \rightarrow V$ is a linear transformation, then V is an $\mathbb{R}[x]$ -module, as described in the book. Is V a free $\mathbb{R}[x]$ -module? Why or why not?

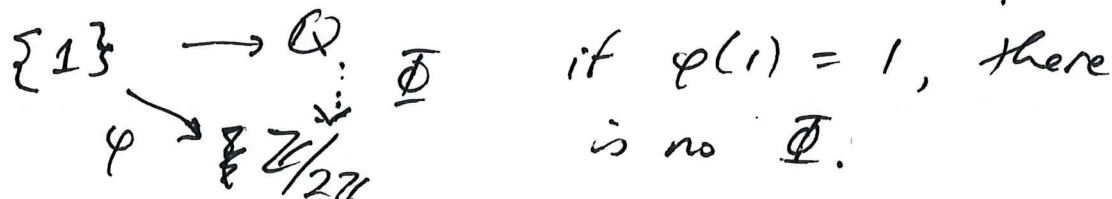
No. Free $\mathbb{R}[x]$ -modules are direct sums of $\mathbb{R}[x]$ with itself, and none of these, except the zero module, ~~are free over $\mathbb{R}$~~ are finite-dimensional over $\mathbb{R}$.

3. Draw the diagram for the "universal property" of the free module $F(A)$ on a set A and explain what the universal property is.



4. Is $\mathbb{Q}$ a free $\mathbb{Z}$ -module on a set consisting of a single element? Why or why not?

No. Existence fails in the universal property.



Thus the ring has order 9, since $a, b \in \mathbb{F}_3$.

When is $a+bx$ invertible mod (x^2) ?

In calculus, you learned that

$$\frac{1}{1-x} = 1+x+x^2+\dots$$

It is also possible to write a series for

$$\frac{1}{a+bx} \quad \text{if } a \neq 0,$$

$$\text{since } a+bx = a\left(1 - \left(-\frac{b}{a}\right)x\right)$$

No worry about convergence since $x^2 = 0$ in

$$\mathbb{F}_3[x]/(x^2).$$

If $a = 0$, $a+bx = bx$ and thus

is either a zero-divisor or zero in

$$\mathbb{F}_3[x]/(x^2).$$

$$x \cdot bx = bx^2 = 0 \pmod{(x^2)}$$

Thus $\left(\mathbb{F}_3[x] / (x^2) \right)^*$ has order 6.

There are 2 groups of order 6: C_6 & S_3 .

The ring is commutative, so answer = C_6 .