Math 525 — Quiz 5 – September 5, 2023

Name:_____

1. Fill in the blank: An ideal I in a commutative ring R is prime iff the quotient R/I is
_____.

2. Fill in the blank: An ideal I in a commutative ring R is maximal iff the quotient R/I
is _____.

3. Give an example of a ring R and two ideals $I, J \subseteq R$ such that I and J are both
maximal but $I \neq J$.

4. Consider the map $\phi\colon \mathbb{R}[x] \to \mathbb{R}$ from the ring of polynomials to the real numbers
defined by $\phi(f) = f(\sqrt{2})$. Is this a ring homomorphism? (Only an answer is necessary.)

5. Consider the "reduction" map $\phi\colon \mathbb{Z} \to \mathbb{Z}/4\mathbb{Z}$.
This reduction map shows that if there is a solution (x, y) to $x^2 + y^2 = 7$ in $\mathbb{Z}$, then there
is a solution in $\mathbb{Z}/4\mathbb{Z}$.
Is there a solution (x, y) to $x^2 + y^2 = 7$ in $\mathbb{Z}/4\mathbb{Z}$? How about in $\mathbb{Z}$?