Math 525 — Quiz 8 – October 2

Name:______________________________

1. Let $\mathbb{F}_2$ be the field with 2 elements and consider the polynomial $p(x) = x^3 + 1 \in \mathbb{F}_2[x]$. Is p irreducible? If so, prove it; if not, give a factorization.

2. Is the following proposition true or false? Explain your answer.

Proposition: A polynomial $f \in \mathbb{Z}[x]$ has a factor of degree one if and only if f has a root in $\mathbb{Z}$.

3. What goes in the blanks in the following statement of Eisenstein's Criterion? Put your answers below the proposition.

Proposition (Eisenstein's Criterion) Let P be a prime ideal of the integral domain R and let $f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial in $R[x]$ ($n \geq 2$). Suppose that $a_{n-1}, \ldots, a_1, a_0$ ____________ and that a_0 ____________. Then $f(x)$ ____________.

4. Show that the polynomial $x^4 + 1$ is irreducible in $\mathbb{Z}[x]$, by substitution of $x + 1$ for x.