Math 525 — Quiz 11 – October 27, 2023

Name:_____

1. The inclusion of the rational numbers into the real numbers makes $\mathbb{R}$ a module over $\mathbb{Q}$. Is $\mathbb{R}$ a free $\mathbb{Q}$-module? Why or why not?

2. If V is a finite dimensional vector space over $\mathbb{R}$ and $T\colon V \to V$ is a linear transformation, then V is an $\mathbb{R}[x]$-module, as described in the book. Is V a free $\mathbb{R}[x]$-module? Why or why not?

3. Draw the diagram for the "universal property" of the free module $F(A)$ on a set A and explain what the universal property is.

4. Is $\mathbb{Q}$ a free $\mathbb{Z}$-module on a set consisting of a single element? Why or why not?