Math 525 — Quiz 13 – November 6

Name:______________________________

The following tensor products are isomorphic to familiar and well-known abelian groups. Using the results about tensor products in Section 10.4, describe the given tensor products in simpler terms, i.e. not as tensor products.

1. $\mathbb{C} \otimes_{\mathbb{R}} (\mathbb{R}^2)$

2. $\mathbb{Q} \otimes_{\mathbb{Z}} (\mathbb{Z}/12\,\mathbb{Z})$

3. $\mathbb{Z}[i] \otimes_{\mathbb{Z}} \mathbb{Q}$

4. $(\mathbb{Z}/10\,\mathbb{Z}) \otimes_{\mathbb{Z}} (\mathbb{Z}/6\,\mathbb{Z})$

5. $\mathbb{Q}(i) \otimes_{\mathbb{Z}[i]} \mathbb{Q}(i)$

6. $(\mathbb{Q}/\mathbb{Z}) \otimes_{\mathbb{Z}} (\mathbb{Q}/\mathbb{Z})$

7. $\mathbb{Z}^5 \otimes_{\mathbb{Z}} \mathbb{Z}^3$

8. The statement "Hom and $\otimes$ are adjoint functors" means that there is a (natural) isomorphism

$$\mathrm{Hom}(U \otimes V, W) \to \mathrm{Hom}(U, \mathrm{Hom}(V, W))$$

How do you define this isomorphism without referring to bases or specific elements of U, V, W? You may, if you like, assume that U, V, W are finite-dimensional vector spaces over the real numbers.