Math 525 — Quiz 15 – November 17

Name:______________________________

1. Let $T\colon V \to V$ be a linear map from a finite-dimensional vector space V to itself, satisfying $T^2 = 0$. Show that $\operatorname{Im}(T) \subset \operatorname{Ker}(T)$.

2. With T as in problem 1, suppose that $\dim V = 5$. What equation is obtained by applying the rank-nullity theorem to T?

3. With T and V as in problem 2, what is the largest possible value of $\dim \operatorname{Im}(T)$?

4. Let $\langle e_1, e_2 \rangle$ be the standard ordered basis for $\mathbb{R}^2$ and let $\phi\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation such that $\phi(e_1) = e_2$ and $\phi(e_2) = e_1$. With respect to the ordered basis $\langle e_1 \otimes e_1, e_1 \otimes e_2, e_2 \otimes e_1, e_2 \otimes e_2 \rangle$ of $\mathbb{R}^2 \otimes_{\mathbb{R}} \mathbb{R}^2$, what is the matrix of $\phi \otimes \phi$?