TOPOLOGICAL CONCORDANCES

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1. Introduction

In this announcement we outline a proof of Hudson's theorem, "Concordance implies isotopy in codimension 3" [3], for topological manifolds. This theorem is also proved by Armstrong [1] in case the ambient manifold is PL, using methods similar to those employed by Hudson. Our approach is to generalize the methods of Morlet [5] to the topological case, using PL approximation theorems locally. We have also generalized Morlet's results on the higher homotopy groups of concordance spaces to the topological case [6]. Detailed proofs will appear elsewhere [7]. I wish to thank professor Lashof at the University of Chicago for many useful discussions and suggestions.

2. Statement of results

Let M and V be topological manifolds. By an embedding $\phi : M \to V$ we understand a locally flat embedding such that $\phi^{-1}(\partial V) = \partial M$ and ϕ meets the boundary regularly. We prove the following theorems:

Theorem 1. *Let V^n be a topological manifold and M^q a compact topological handlebody. Let*

$$\phi : (M \times I, M \times 0, M \times 1) \to (V \times I, V \times 0, V \times 1)$$

be an embedding. Denote $\phi | M \times 0$ by g and assume

$$\phi | \partial M \times I = (g | \partial M) \times 1) I$$

(∂M may be empty) and $n - q \geq 3$, $n \geq 5$. If $n = 5$ we further assume that ∂V is a stable manifold. Then there is an ambient isotopy H^t of $V \times I$, fixing $\partial V \times I \cup V \times 0$ such that $H^1 \circ \phi = g \times 1_I$.

Theorem 2. *Let V^n be a topological manifold and (K, K') a finite polyhedral pair,* $\dim(K - K') = q$ *(K' may be empty). Let*

$$\phi : (K \times I, K \times 0, K \times 1) \to (V \times I, V \times 0, V \times 1)$$

be a locally nice embedding such that $\phi^{-1}(\partial V \times I) = K' \times I$. Denote $\phi | V \times 0$ by g and assume ϕ is equal to $g \times 1_I$ in a neighborhood of $K' \times I$. Also assume $n - q \geq 3$ and $n \geq 5$. If $n = 5$ we assume ∂V is a stable manifold. Then there is an ambient isotopy H^t of $V \times I$ fixing $\partial V \times I \cup V \times 0$ such that $H^1 \circ \phi = g \times 1_I$.

3. Outline of proof

First we prove Theorem 1 for M a disc. So consider a concordance $\phi : D^q \times I \to V \times I$ and denote $\phi(D^q \times I)$ by L_1 and $g(D^q) \times I$ by L_0.

(1) We find PL product neighborhood U of L_0. Using PL approximation theorems we can then isotop L_1 and shrink U such that $U \cap L_1 \subset U$ is a PL embedding. We can further isotop L_1 such that L_1 intersects L_2 blocktransversally, and both these isotopies can be made such that the restriction to $\phi(\partial D^q \times I) = g(\partial D^q) \times I$ is a product isotopy.

(2) Having moved L_1 to intersect L_0 block-transversally, denote $X = L_0 \cap L_1$, $X^0 = X \cap V \times 0$ and $X^1 = X \cap V \times 1$. We define $\tau(X) \geq \alpha$ if X is obtained from $X^0 \times I$ by adding handles not meeting ∂X^0 of dimension j, $\alpha \leq j \leq \dim(X) - \alpha$. Assume $\tau(X) \geq \alpha - 1$ and let $\mu_s : D^{\alpha-1} \to X$ be the cores of the $\alpha - 1$ handles relative to X^0 and X^1. μ_s can be bounded by embedded discs in L_0 as well as in L_1, and the union of the bounding discs can be bounded in $V \times I - L_0 \cup L_1$ by an embedded disc $F_2(D^{\alpha+1}$. We prove this using homotopy-theoretic considerations and Less' immersion theorem [4]. Now we slide L_1 across $F_s(D^{\alpha+1})$ (analogously to the standard Whitney trick for deletion of double points) so as to remove a regular neighborhood of $\mu_s(D^{\alpha-1})$. We thus obtain that $\tau(\text{resulting } X) \geq \alpha$.

(3) We inductively obtain that $\tau(X) > \frac{1}{2}\dim(X)$, so X is a product. This product structure extends to $L_0 \cup L_1$ and further to $V \times I$ (using the topological s-cobordism theorem). Using this product structure we move L_1 back so that its image finally coincides with L_0, and we can then use an Alexander isotopy to straighten L_1.

(4) The isotopy we obtained above does not fix $\partial D^q \times I \cup D^q \times 0$, but it may be modified to do so by using a collar of the boundary. We finally obtain an ambient isotopy using the isotopy extension theorem [2]

The general case follows from that of a disc by induction on skeleta. First we straighten the 0-skeleton. The we straighten a neighborhood of the 0-skeleton using local PL approximation and Hudson's theorem. Cutting out a regular neighborhood of the 0-skeleton makes the 1-skeleton into a disjoint union of 1-discs which can be straightened from what we already have proved and, we proceed inductively.

References

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