SMOOTHING H-SPACES

ERIK KJÆR PEDERSEN

Consider a partition of all primes $\Pi = P_1 \cup \cdots P_k$, and let G_i be quasi-finite complexes such that $(G_i)_{P_i}$, G_i localized at P_i are H-spaces, that are rationally equivalent as H-spaces. The homotopy pullback

$$\begin{array}{ccc}
(G_1)_{P_1} & \longrightarrow & (G_1)_0 \\
 & & \downarrow \cong \\
(G_2)_{P_2} & \longrightarrow & (G_2)_0 \\
 & & \downarrow \cong \\
 & & \vdots \\
 & & \downarrow \cong \\
(G_k)_{P_k} & \longrightarrow & (G_k)_0
\end{array}$$

is well known to be a quasifinite H-space [1]. We will say that X is obtained by Zabrodsky mixing of $G_1, \cdots, G_k$ at $P_1, \cdots, P_k$. Note that X depends on the rational equivalences chosen.

We consider H-spaces obtained by mixing the following types of spaces:

At the prime 2*:* products of Lie groups and S^7 and RP^7.

At odd primes: products of

(a) Lie groups, S^7, RP^7 and
(b) any stably reducible quasi-finite P. D. space with no 3-dimensional generator of the rational exterior algebra cohomology and finitely presented fundamental group, and
(c) principal S^3-bundles with basespace a stably reducible quasi-finite P. D. space

We prove the following theorems:

THEOREM. *Let X be as above,* 1*-connected; then X is of the homotopy type of a parallellizable differentiable manifold.*

We notice that by choosing all G_i equal, but choosing different rational equivalences we get the following:

COROLLARY. *Let Y be in the genus of a simply connected Lie group G (i. e. $Y_p \cong G_p$ for all primes p); then Y is homotopy equivalent to a parallellizable manifold.*

In the nonsimply connected context we prove the following:

THEOREM. *Let X be a as above; then X is of the homotopy type of a finite complex, and in the genus of X is a parallellizable differentiable manifold.*

Indication of proof. The case where $X_{(2)} \cong (RP^3)^k \times (S^7)^l \times (RP^7)^M$ requires special arguments. In all other cases we proceed by constructing a fibration $S^1 \to X \to Y$, where Y is a stably reducible P. D. space. The map p induces isomorphisms on fundamental groups, and the fibration is orientable. We then use [2] to compute the Wall finiteness obstruction for X. The formula in this case says $\sigma(X) = \chi((S^1) \cdot \sigma(Y)$, χ the euler characteristic; hence $\sigma(X) = 0$, so X is homotopy equivalent to a finite CW complex.

We then consider X as a P. D. boundary of the corresponding D^2-fibration $D^2 \to E \to Y$, and notice that the classifying map $E \to BG$ reduces to BO since Y is stably reducible and S^1-fibrations are equivalent to $O(2)$-bundles. This allows us to set up a surgery problem

$$(M.\partial M) \to (E, Y)$$

and we proceed to show the surgery problem $\partial M \to Y$ has obstruction 0. The reduction of E is trivial when restricted to Y; hence Y is of the homotopy type of a parallellizable differentiable manifold.

REFERENCES

[1] P. Hilton, G. Mislin, and J. Roitberg, *Localization of Nilpotent Groups ands Spaces*, North Holland Mathematics studies, vol. 15, North - Holland Publishing co., Amsterdam - New York, 1975.

[2] E. K. Pedersen and L. Taylor, *The Wall finiteness obstruction for a fibration*, Amer. J. Math. **100** (1978), 887–896.

ODENSE UNIVERSITY