The Atiyah-Singer Index Theorem I

Paul Loya

Main idea

Definition

in·dex
n. pl. **in·dex·es** or **in·di·ces**

1. Something that serves to guide, point out, or otherwise facilitate reference.
2. Something that reveals or indicates; a sign.
3. An indicator or pointer, as on a scientific instrument.
4. Mathematics: A number derived from a formula, used to characterize a set of data.

Today we'll study two indexes.

Outline of talk: Five main points

I. The index of a space (Euler characteristic)

Outline of talk: Five main points

I. The index of a space (Euler characteristic)

II. The index of a linear map

Outline of talk: Five main points

I. The index of a space (Euler characteristic)

II. The index of a linear map

III. The Gauss-Bonnet theorem — traditional version

Outline of talk: Five main points

I. The index of a space (Euler characteristic)

II. The index of a linear map

III. The Gauss-Bonnet theorem — traditional version

IV. The Gauss-Bonnet theorem — index version

Outline of talk: Five main points

I. The index of a space (Euler characteristic)

II. The index of a linear map

III. The Gauss-Bonnet theorem — traditional version

IV. The Gauss-Bonnet theorem — index version

V. The Atiyah-Singer index theorem

I. The Euler characteristic

Preview of Part I

- The Euler Characteristic is an integer obtained by taking the *difference* of two numbers.
- The Euler Characteristic is a topological invariant.

I. The Euler characteristic

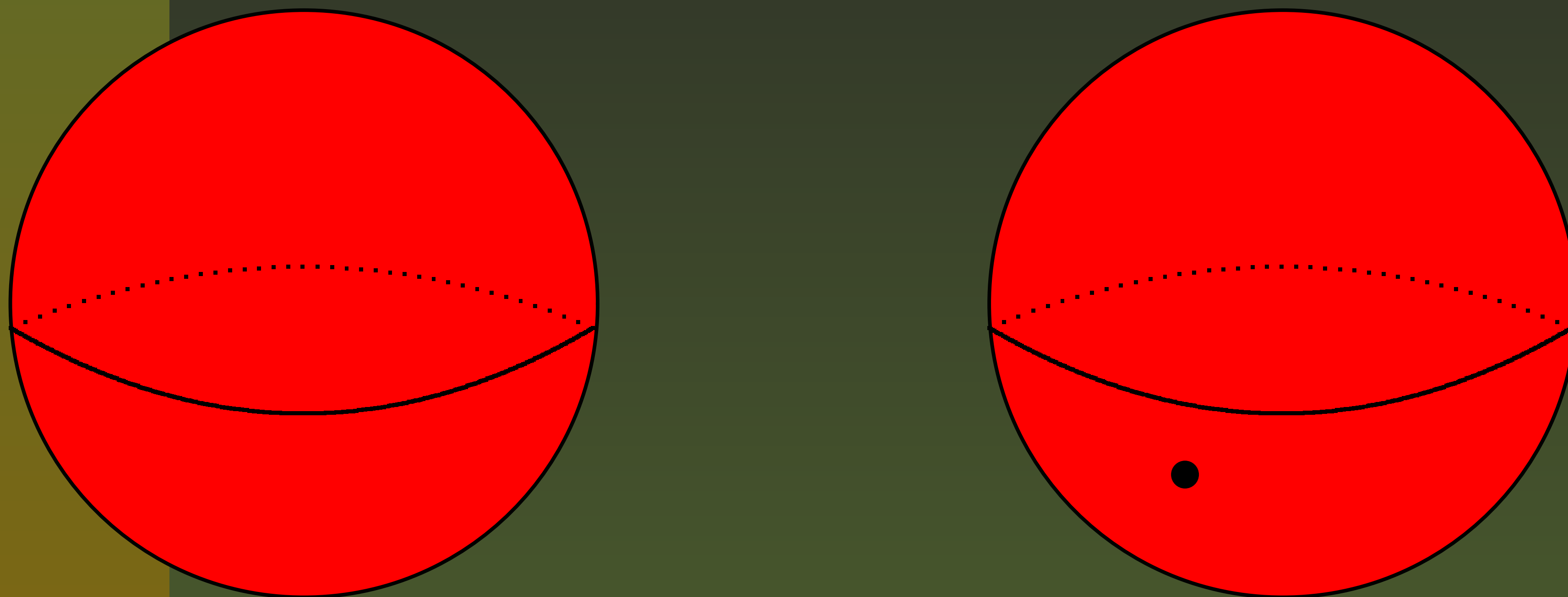
The **Euler Characteristic** of a compact space S (really, a CW complex of dimension ≤ 2) is obtained by

1. putting dots all over your object.
2. connecting the dots (no crossings and every dot must be linked to any other one through lines).
3. Counting the number of even (0 or 2) dimensional shapes formed (E).
4. Counting the number of odd (1) dimensional shapes formed (O).

$$\boxed{\chi(S) = E - O.}$$

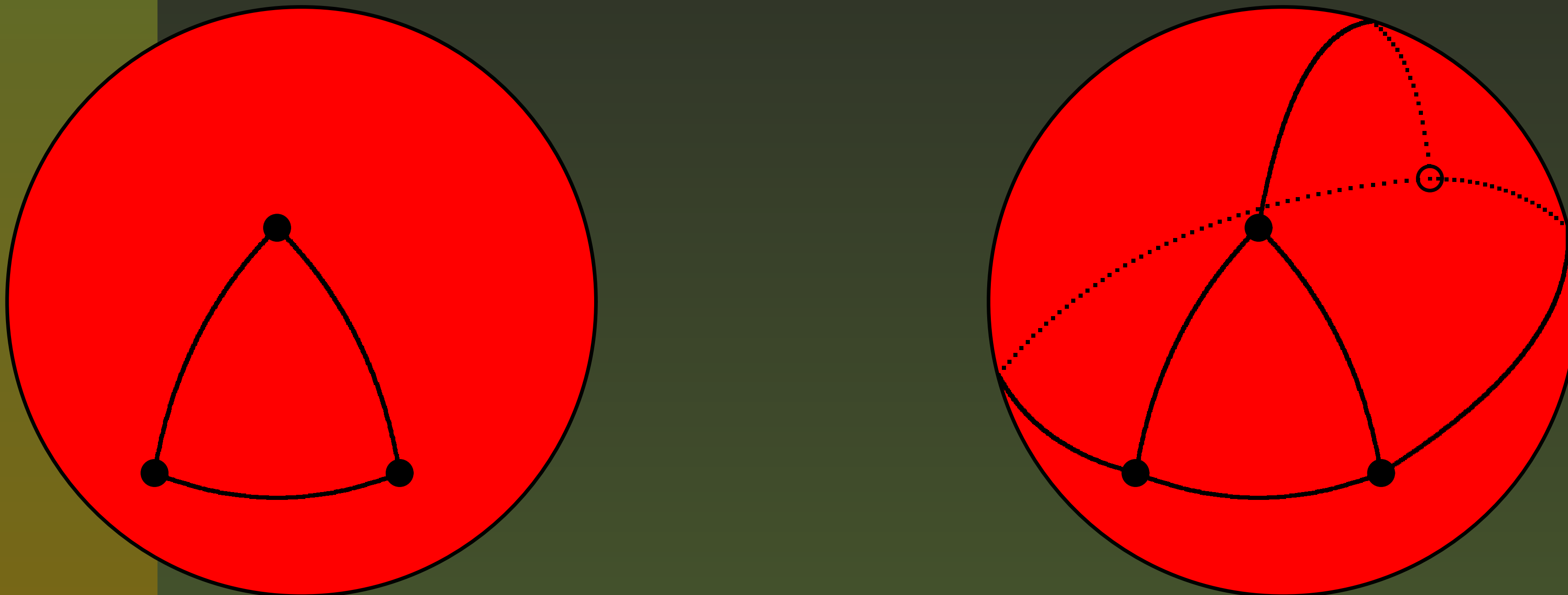
I. The Euler characteristic

The sphere:

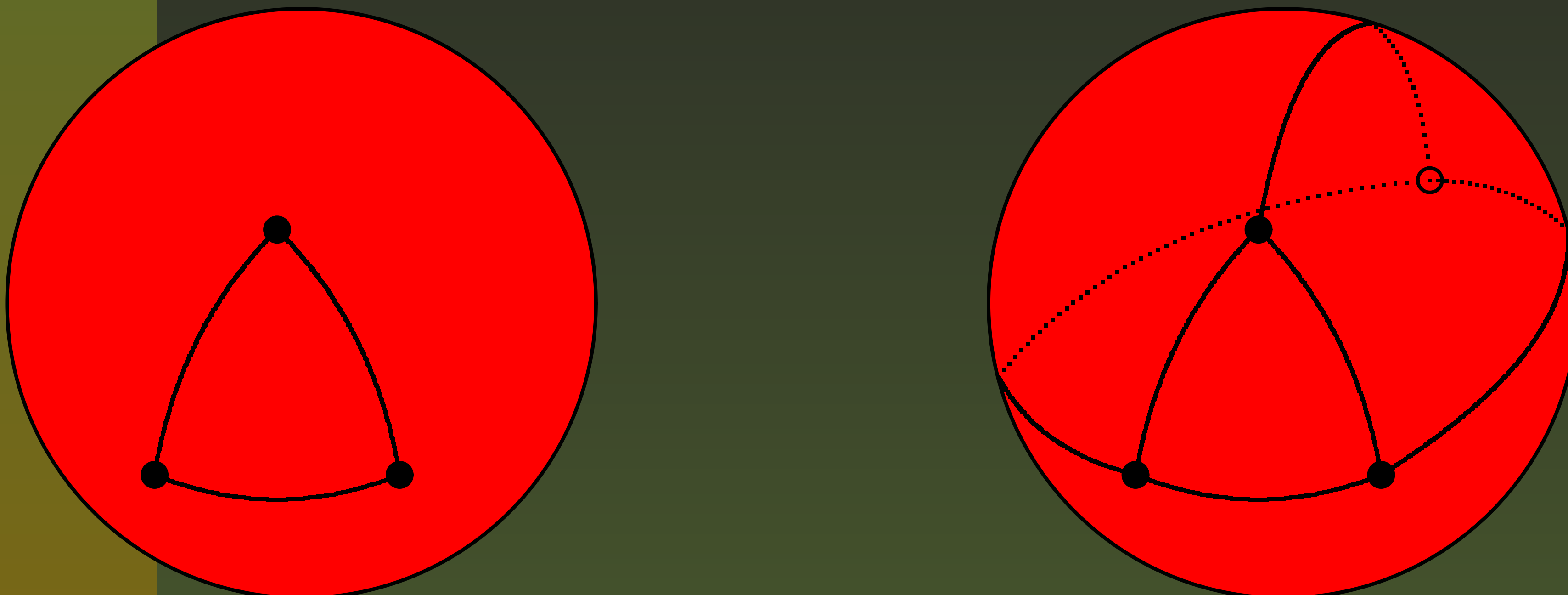


$$\chi(\mathbb{S}^2) = E - O = (1 + 1) - 0 = 2.$$

I. The Euler characteristic



I. The Euler characteristic

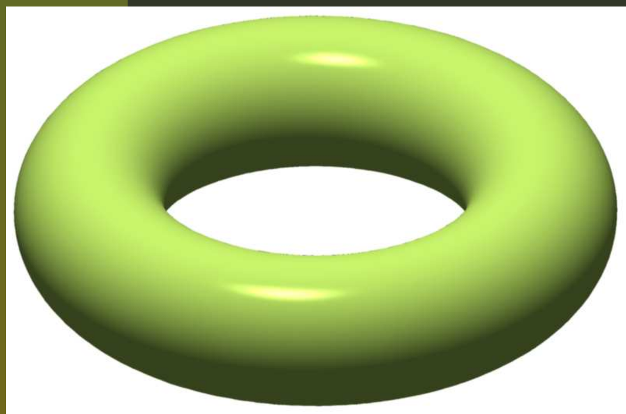


$$\chi(\mathbb{S}^2) = E - O = (3+2) - 3 = 2.$$

$$\chi(\mathbb{S}^2) = E - O = (4+4) - 6 = 2.$$

No matter how many dots you mark and how you connect them, you get 2.

I. The Euler characteristic



$$\chi(\text{torus}) = 0,$$

$$\chi(\text{double torus}) = -2,$$

$$\chi(\text{triple torus}) = -4.$$

I. The Euler characteristic

- In what sense is the Euler Characteristic an index?

It indicates, reveals, characterizes the topology.

I. The Euler characteristic

- In what sense is the Euler Characteristic an index?

It indicates, reveals, characterizes the topology.

Theorem 1: Two compact surfaces in $\mathbb{R}^3$ are homeomorphic if and only if their Euler characteristics are equal.

In other words, the EC is “stable” — if you deform a shape the EC remains the same.

I. The Euler characteristic

- In what sense is the Euler Characteristic an index?

It indicates, reveals, characterizes the topology.

Theorem 1: Two compact surfaces in $\mathbb{R}^3$ are homeomorphic if and only if their Euler characteristics are equal.

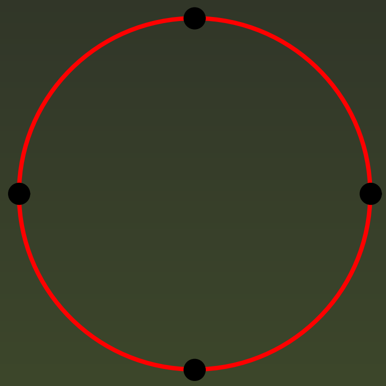
In other words, the EC is "stable" — if you deform a shape the EC remains the same.

Theorem 2: For a compact surface S in $\mathbb{R}^3$,

$$\chi(S) = 2 - 2 \times (\text{the number of holes of } S).$$

How about 1-dimensional examples?

I. The Euler characteristic



$$\chi(\mathbb{S}^1) = 4 - 4 = 0.$$

I. The Euler characteristic

$$\chi(\mathbb{S}^1) = 4 - 4 = 0.$$

$$\chi(\mathbb{R}) = 4 - 3 = 1.$$

(For unbounded spaces we have to throw away unbounded parts.)

I. The Euler characteristic

Summary of Part I

- The Euler characteristic of a space S is

$$\chi(S) = E - O,$$

a *difference* of two integers.

- The Euler characteristic is "stable" — it's a topological invariant.

- Examples:

2-dim: $\chi(\mathbb{S}^2) = 2$ and $\chi(\mathbb{T}) = 0.$

1-dim: $\chi(\mathbb{S}^1) = 0$ and $\chi(\mathbb{R}) = 1.$

II. The index of a linear map

Preview of Part II

- The index of a linear map is an integer obtained by taking the *difference* of two numbers.

- The index is an “analytic” invariant.

II. The index of a linear map

Let V and W be finite-dimensional vector spaces and let

$$L : V \to W$$

be a linear map. **What is a good notion of “index”?**

II. The index of a linear map

Let V and W be finite-dimensional vector spaces and let

$$L : V \to W$$

be a linear map. **What is a good notion of "index"?**

What are some important objects associated to L?

- Kernel: $\ker L = \{v \in V \mid Lv = 0\}$.
- Cokernel: $W/\mathrm{Im}L$ (quotient vector space) where $\mathrm{Im}L = \{w \in W \mid w = Lv$ some $v \in V\} \subseteq W$.

$\mathrm{coker}\, L$ represents all vectors in W not in the image of L.

How do you get an "index"?

II. The index of a linear map

Definition:

$$\boxed{\operatorname{ind} L = \dim(\ker L) - \dim\left(W/\mathrm{Im}L\right).}$$

- Remark: We copy the Euler Characteristic, which is also a difference of two integers.

II. The index of a linear map

Definition:

$$\boxed{\operatorname{ind} L = \dim(\ker L) - \dim\left(W/\mathrm{Im}L\right).}$$

- Remark: We copy the Euler Characteristic, which is also a difference of two integers.

Theorem: For any linear map $L : V \to W$, between finite-dimensional vector spaces, we have

$$\operatorname{ind} L = \dim V - \dim W.$$

II. The index of a linear map

Proof: By the rank-nullity theorem,

$$\dim \ker L + \dim \operatorname{Im} L = \dim V.$$

II. The index of a linear map

Proof: By the rank-nullity theorem,

$$\dim \ker L + \dim \operatorname{Im} L = \dim V.$$

Therefore,

$$\dim \ker L + \dim \operatorname{Im} L - \dim W = \dim V - \dim W,$$

or

$$\dim \ker L - \big(\dim W - \dim \operatorname{Im} L \big) = \dim V - \dim W.$$

II. The index of a linear map

Proof: By the rank-nullity theorem,

$$\dim\ker L + \dim\operatorname{Im} L = \dim V.$$

Therefore,

$$\dim\ker L + \dim\operatorname{Im} L - \dim W = \dim V - \dim W,$$

or

$$\dim\ker L - \big(\dim W - \dim\operatorname{Im} L\big) = \dim V - \dim W.$$

Therefore,

$$\dim\ker L - \dim\big(W/\operatorname{Im} L\big) = \dim V - \dim W.$$

II. The index of a linear map

- Some thought reveals a “deep” connection:

$$\boxed{\begin{array}{ccc} \operatorname{ind} L & \Longrightarrow\!\!= & \dim V - \dim W \\ \textbf{linear algebraic} & & \textbf{topological} \end{array}}$$

This is a “deep” result.

Therefore, the index is “stable” — if you change the linear map, the index remains the same.

What about infinite-dimensional vector spaces?

II. The index of a linear map

Let $L : V \to W$ be a linear map between two possibly infinite-dimensional vector spaces.

(Same) **Definition:**

$$\boxed{\operatorname{ind} L = \dim(\ker L) - \dim\Big(W/\mathrm{Im}L\Big).}$$

$\operatorname{ind} L$ is only defined when the dimensions on the RHS are finite . . . otherwise $\operatorname{ind} L$ is undefined.

Linear maps for which $\operatorname{ind} L$ is defined are called **Fredholm**. The index, it turns out, is still “stable” when V and W are infinite-dimensional.

What does Fredholm mean intuitively?

II. The index of a linear map

Fredholm means:

$$1) \quad \dim(\ker L) < \infty \quad \text{and} \quad 2) \quad \dim\left(W/\mathrm{Im}L\right) < \infty$$

II. The index of a linear map

Fredholm means:

$$1) \quad \dim(\ker L) < \infty \quad \text{and} \quad 2) \quad \dim\Big(W/\mathrm{Im} L\Big) < \infty$$

Recall:

L is injective $\iff \ker L = 0.$

L is surjective $\iff \mathrm{Im}\, L = W$; that is, $W/\mathrm{Im}\, L = 0.$

II. The index of a linear map

Fredholm means:

$$1)\ \ \dim(\ker L) < \infty \quad \text{and} \quad 2)\ \ \dim\left(W/\mathrm{Im}L\right) < \infty$$

Recall:
L is injective $\iff \ker L = 0$.
L is surjective $\iff \mathrm{Im}\, L = W$; that is, $W/\mathrm{Im}\, L = 0$.

Conclusion:
Fredholm conveys that L is

$1)$ "almost" injective and $2)$ "almost" surjective.

That is, L is "almost" an isomorphism.

What does $\mathrm{ind}\, L$ **indicate, reveal, characterize?**

II. The index of a linear map

Write $V = \ker L \oplus V'$, $W = W' \oplus \operatorname{Im} L$.

II. The index of a linear map

Write $V = \ker L \oplus V'$, $W = W' \oplus \operatorname{Im} L$.

$$L : \begin{array}{c} \ker L \\ \oplus \\ V' \end{array} \longrightarrow \begin{array}{c} W' \\ \oplus \\ \operatorname{Im} L \end{array} \qquad \big(\text{NB: } W' \cong W/\operatorname{Im} L\big)$$

Can we make make L invertible by changing L on its kernel?

II. The index of a linear map

Write $V = \ker L \oplus V'$, $W = W' \oplus \operatorname{Im} L$.

$$L : \begin{array}{c} \ker L \\ \oplus \\ V' \end{array} \longrightarrow \begin{array}{c} W' \\ \oplus \\ \operatorname{Im} L \end{array} \qquad \left(\text{NB: } W' \cong W/\operatorname{Im} L\right)$$

Can we make make L invertible by changing L on its kernel?

Cases:

$$\operatorname{ind} L = 0 \quad \Longrightarrow \quad \text{can make } L \text{ invertible}$$

$$\operatorname{ind} L > 0 \quad \Longrightarrow \quad \text{excess of null vectors}$$

$$\operatorname{ind} L < 0 \quad \Longrightarrow \quad \text{deficiency of null vectors}$$

II. The index of a linear map

Conclusion:

- $\operatorname{ind} L$ indicates, reveals, characterizes how "far" L is from being made invertible.

I.e., we can modify L on $\ker L$ to get an invertible operator iff $\operatorname{ind} L = 0$.

The further $\operatorname{ind} L$ is from zero, the more "non-invertible" L is.

Examples of Fredholm operators?

II. The index of a linear map

Ex 1: Let $V = W = C^\infty(\mathbb{R})$ and let $L = \dfrac{d}{dx} : V \to W$.

(That is, $L(f) = f'$.)

II. The index of a linear map

Ex 1: Let $V = W = C^\infty(\mathbb{R})$ and let $L = \dfrac{d}{dx} : V \to W$.

(That is, $L(f) = f'$.)

- $\ker L$:

$$\ker L = \{f \in C^\infty(\mathbb{R}) \mid f' = 0\} = \text{const. functions} \cong \mathbb{R}.$$

Thus, $\dim(\ker L) = 1$.

II. The index of a linear map

Ex 1: Let $V = W = C^\infty(\mathbb{R})$ and let $L = \dfrac{d}{dx} : V \to W$.

(That is, $L(f) = f'$.)

- $\ker L$:

$$\ker L = \{f \in C^\infty(\mathbb{R}) \mid f' = 0\} = \text{const. functions} \cong \mathbb{R}.$$

Thus, $\dim(\ker L) = 1$.

- $W/\operatorname{Im} L$: Claim: $\operatorname{Im} L = W$. Let $g \in W = C^\infty(\mathbb{R})$.

Define

$$f(x) = \int_0^x g(t)\, dt.$$

Then $f \in V$ and $L(f) = g$. Thus,

$$\dim(W/\operatorname{Im} L) = \dim(W/W) = \dim(0) = 0.$$

II. The index of a linear map

Ex 1 con't:

Thus,

$$\operatorname{ind} L = \dim(\ker L) - \dim\left(W/\mathrm{Im}L\right) = 1 - 0 = 1.$$

II. The index of a linear map

Ex 1 con't:
Thus,

$$\operatorname{ind} L = \dim(\ker L) - \dim\left(W/\mathrm{Im}L\right) = 1 - 0 = 1.$$

- Hence,

$$\boxed{\begin{array}{ccc} \operatorname{ind} L & = & \chi(\mathbb{R}) \\ \textbf{analytical} & & \textbf{topological} \end{array}}$$

This is a "deep" result.

How about one more example?

II. The index of a linear map

Ex 2: Let $V = W = C^\infty(\mathbb{S}^1)$ and let

$$L = \frac{d}{d\theta} : V \to W.$$

II. The index of a linear map

Ex 2: Let $V = W = C^\infty(\mathbb{S}^1)$ and let

$$L = \frac{d}{d\theta} : V \to W.$$

- $\ker L$:

$\ker L = \{f \in C^\infty(\mathbb{S}^1) \mid f'(\theta) = 0\} =$ const. functions $\cong \mathbb{R}$

Thus,

$$\dim(\ker L) = 1.$$

II. The index of a linear map

Ex 2 con't:

Lemma: $h \in \operatorname{Im} L \iff \displaystyle\int_0^{2\pi} h(\theta)\, d\theta = 0.$

II. The index of a linear map

Ex 2 con't:

Lemma: $h \in \operatorname{Im} L \iff \int_0^{2\pi} h(\theta)\, d\theta = 0.$

Proof: Suff: Assume $h = Lf = f'(\theta)$. Then,

$$\int_0^{2\pi} h(\theta)\, d\theta = \int_0^{2\pi} f'(\theta)\, d\theta = f(2\pi) - f(0) = 0.$$

II. The index of a linear map

Ex 2 con't:

Lemma: $h \in \operatorname{Im} L \iff \int_0^{2\pi} h(\theta)\, d\theta = 0.$

Proof: Suff: Assume $h = Lf = f'(\theta)$. Then,

$$\int_0^{2\pi} h(\theta)\, d\theta = \int_0^{2\pi} f'(\theta)\, d\theta = f(2\pi) - f(0) = 0.$$

Nec: Assume $\int_0^{2\pi} h(\theta)\, d\theta = 0$. Define

$$f(\theta) = \int_0^{\theta} h(t)\, dt.$$

Then $f(0) = f(2\pi) = 0$. Therefore, $f \in C^\infty(\mathbb{S}^1)$. Moreover,

$$f'(\theta) = h(\theta) \quad \Longrightarrow \quad h = Lf \in \operatorname{Im} L.$$

II. The index of a linear map

Ex 2 con't:

- $W/\operatorname{Im} L$: Let $g \in W = C^\infty(\mathbb{S}^1)$. Define

$$h := g - \frac{1}{2\pi}\int_0^{2\pi} g(t)\,dt.$$

Then, $\int_0^{2\pi} h(\theta)\,d\theta = 0$.

II. The index of a linear map

Ex 2 con't:

- $W/\operatorname{Im} L$: Let $g \in W = C^\infty(\mathbb{S}^1)$. Define

$$h := g - \frac{1}{2\pi}\int_0^{2\pi} g(t)\,dt.$$

Then, $\int_0^{2\pi} h(\theta)\,d\theta = 0$.

$$\text{Lemma} \Longrightarrow h = Lf \ \text{(some } f\text{)} \Longrightarrow g = Lf + \text{const.}$$
$$\Longrightarrow W/\operatorname{Im} L \cong \mathbb{R}.$$

Thus,

$$\dim(W/\operatorname{Im} L) = \dim(\mathbb{R}) = 1.$$

II. The index of a linear map

Ex 2 con't:

Thus,

$$\operatorname{ind} L = \dim(\ker L) - \dim\left(W/\mathrm{Im}L\right) = 1 - 1 = 0.$$

II. The index of a linear map

Ex 2 con't:

Thus,

$$\operatorname{ind} L = \dim(\ker L) - \dim\left(W/\mathrm{Im}L\right) = 1 - 1 = 0.$$

- Hence,

$$\boxed{\begin{array}{ccc} \operatorname{ind} L & = & \chi(\mathbb{S}^1) \\ \textbf{analytical} & & \textbf{topological} \end{array}}$$

This is a "deep" result.

II. The index of a linear map

Summary of Part II

- For a linear map $L : V \to W$,

$$\operatorname{ind} L = \dim(\ker L) - \dim\Big(W/\mathrm{Im}L\Big),$$

provided that the RHS is well-defined as an integer.

- $\operatorname{ind} L$ indicates, reveals, characterizes how far L is from being made invertible.

- Specific examples suggest that $\operatorname{ind} L$ indicates, reveals, characterizes certain topological information. The index is “stable” — it’s an analytic invariant.

- The interplay of *analysis* and *topology* through the index is “deep” because it relates two distinct notions.

III. The Gauss-Bonnet theorem

Preview of Part III

- The Gauss-Bonnet theorem gives a formula relating two aspects of a surface: The *topology* and the *geometry*.

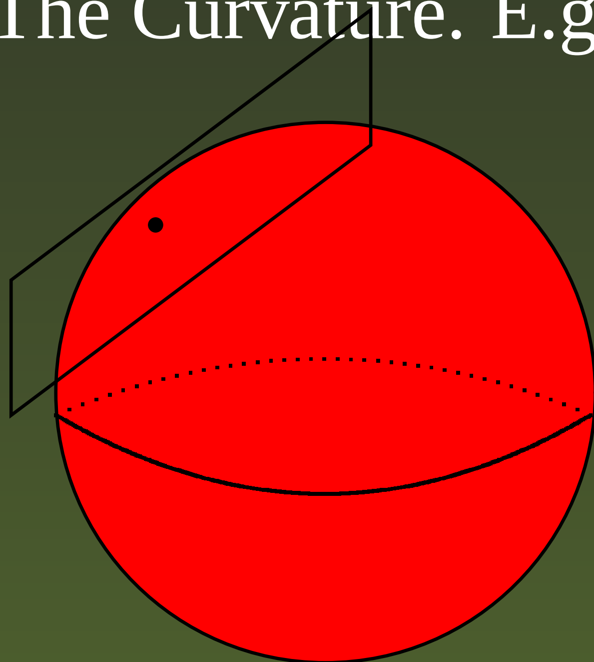
III. The Gauss-Bonnet theorem

- Topology: The Euler Characteristic. E.g. Recall that $\chi(\mathbb{S}^2) = 2$.

III. The Gauss-Bonnet theorem

- Topology: The Euler Characteristic. E.g. Recall that $\chi(\mathbb{S}^2) = 2$.

- Geometry: The Curvature. E.g. consider $\mathbb{S}^2$,



$K = 1$

The curvature measures how much the surface bends away from the tangent plane. $K > 0$ "bending away"; $K < 0$ "bending into".

III. The Gauss-Bonnet theorem

- Observation:

$$\frac{1}{2\pi}\int_{\mathbb{S}^2} K = \frac{1}{2\pi}\int_{\mathbb{S}^2} 1 = \frac{1}{2\pi}\mathrm{Area}(\mathbb{S}^2)$$

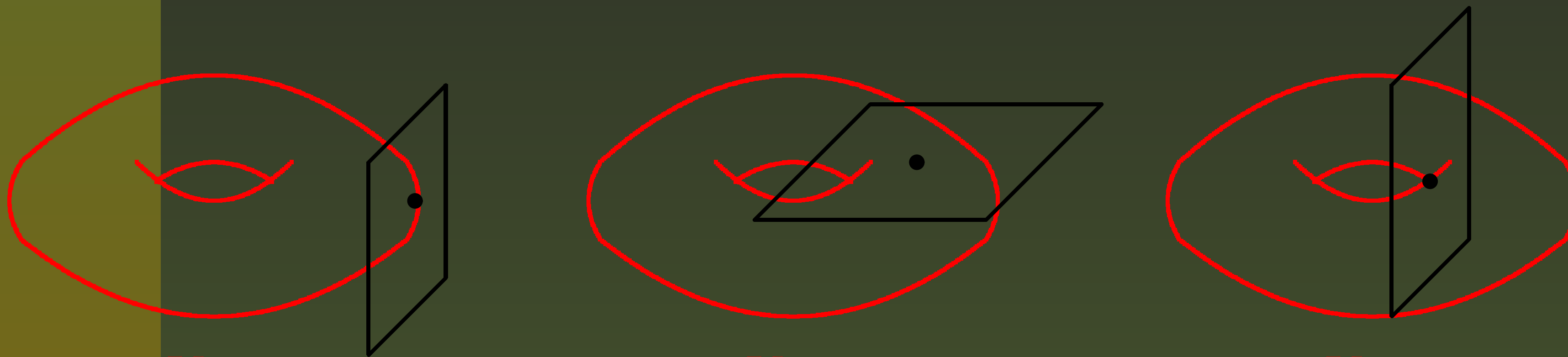
$$= \frac{1}{2\pi}(4\pi) = 2.$$

- Therefore,

$$\boxed{\chi(\mathbb{S}^2) = \frac{1}{2\pi}\int_{\mathbb{S}^2} K.}$$

III. The Gauss-Bonnet theorem

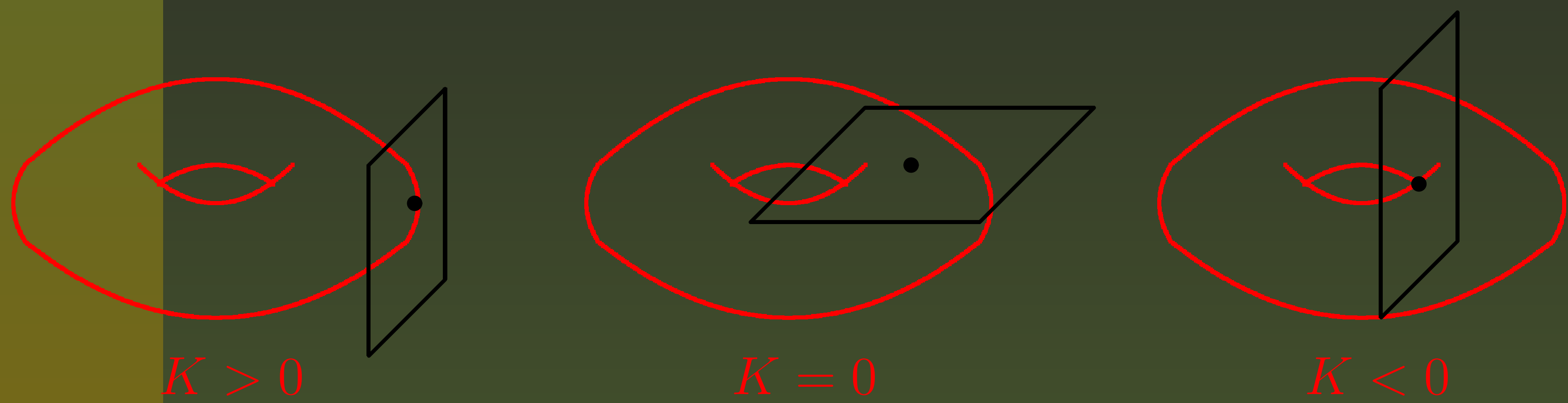
Another example: The Torus $\mathbb{T}$. Recall $\chi(\mathbb{T}) = 0$.



$K > 0$ $\qquad$ $K = 0$ $\qquad$ $K < 0$

III. The Gauss-Bonnet theorem

Another example: The Torus $\mathbb{T}$. Recall $\chi(\mathbb{T}) = 0$.



We have

$$\frac{1}{2\pi}\int_{\mathbb{T}} K = 0.$$

- Therefore,

$$\boxed{\chi(\mathbb{T}) = \frac{1}{2\pi}\int_{\mathbb{T}} K.}$$

III. The Gauss-Bonnet theorem

Gauss-Bonnet theorem: Given an oriented, compact, 2-dimensional, Riemannian manifold M, we have

$$\boxed{\begin{array}{ccc} \chi(M) & = & \frac{1}{2\pi}\int_M K \\ \textbf{topological} & & \textbf{geometrical} \end{array}}$$

- The G-B formula bridges two areas of math: topology and geometry.

- The G-B formula is a “deep” result because it relates two seemingly distinct properties of a surface.

III. The Gauss-Bonnet theorem

Ex. Each of the surfaces

satisfies

$$\frac{1}{2\pi}\int_M K = 2.$$

Not at all obvious!

III. The Gauss-Bonnet theorem

Summary of Parts I, II, III

- The Euler Characteristic is a topological invariant.

- The index of a linear map is an analytical invariant and for one-dimensional topological spaces (the line and the circle), we saw that

$$\operatorname{ind} L = \text{Euler Characteristic.}$$

- The Gauss-Bonnet formula: For a two-dimensional surface,

$$\text{Euler Characteristic} = \frac{1}{2\pi}\int_M K.$$

IV. Index version of Gauss-Bonnet

Preview of Part IV

- As expected, given two-dimensional surface, there is a differential operator L such that

$$\operatorname{ind} L = \text{Euler Characteristic} = \frac{1}{2\pi}\int_M K.$$

IV. Index version of Gauss-Bonnet

Let M be an oriented, compact, 2-dim., Riemannian manifold.

- Vector spaces:

$$
\begin{aligned}
V &= C^\infty(M, TM) \\
&= \text{infinitely differentiable tangent vector fields on } M
\end{aligned}
$$

$$
\begin{aligned}
W &= C^\infty(M, \mathbb{R}^2) \\
&= \text{(basically) pairs of infinitely differentiable functions on } M
\end{aligned}
$$

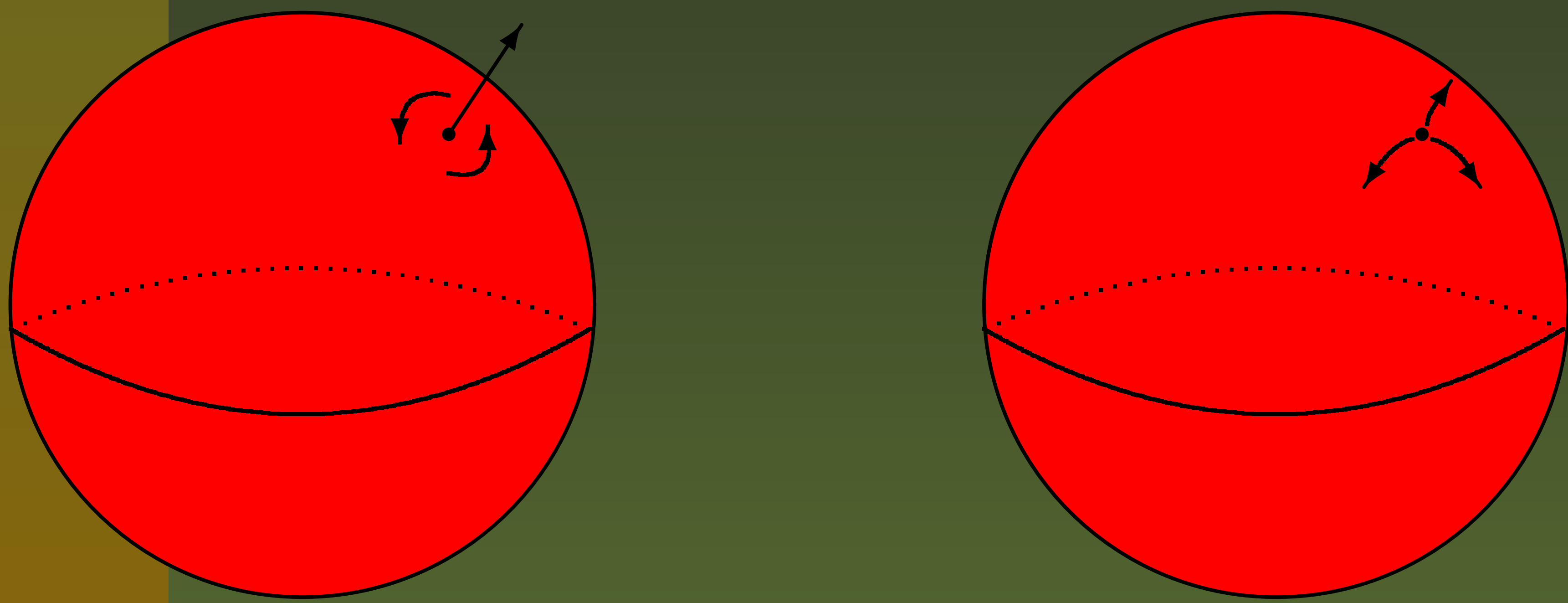
"TM" is for "tangent bundle of M".

IV. Index version of Gauss-Bonnet

- Linear map: $L : C^\infty(M, TM) \to C^\infty(M, \mathbb{R}^2)$ is the map

$$L(v) = (-\mathrm{curl}\, v\ ,\ \mathrm{div}\, v).$$

L is called the **poor man's Gauss-Bonnet operator**.



IV. Index version of Gauss-Bonnet

Exercise: Think of the torus as $\mathbb{T} = \mathbb{S}^1_\theta \times \mathbb{S}^1_\varphi$. Let

$$\operatorname{curl}\left(f\vec{\imath}_\theta + g\vec{\jmath}_\varphi\right) = \frac{\partial g}{\partial \theta} - \frac{\partial f}{\partial \varphi}.$$

$$\operatorname{div}\left(f\vec{\imath}_\theta + g\vec{\jmath}_\varphi\right) = \frac{\partial f}{\partial \theta} + \frac{\partial g}{\partial \varphi}.$$

Prove that

$$\operatorname{ind} L = 0.$$

Recall that $\chi(\mathbb{T}) = 0 \quad \Longrightarrow \quad \operatorname{ind} L = \chi(\mathbb{T})$.

IV. Index version of Gauss-Bonnet

- **Theorem:** (Consequence of Hodge theory). We have

$$\operatorname{ind} L = \chi(M).$$

This theorem is a generalization of the $\mathbb{R}$ and $\mathbb{S}^1$ examples we did earlier!

- Since $\chi(M) = \frac{1}{2\pi}\int_M K$, we have

$$\boxed{\begin{array}{ccc} \operatorname{ind} L & = & \frac{1}{2\pi}\int_M K \\ \textbf{analytical} & & \textbf{geometrical} \end{array}}$$

This is a "deep" result.

IV. Index version of Gauss-Bonnet

Summary of Part IV

If $L : C^\infty(M, TM) \to C^\infty(M, \mathbb{R}^2)$ is the poor man's G-B operator $(-\mathrm{curl}\ ,\ \mathrm{div}\ )$, then

$$\boxed{\begin{array}{ccc} \mathrm{ind}\, L & = & \frac{1}{2\pi} \int_M K \\ \textbf{analytical} & & \textbf{geometrical} \end{array}}$$

This is the index formula version of the Gauss-Bonnet formula.

The Atiyah-Singer index formula is a higher-dimensional version of the above formula!

Part IV: The Atiyah-Singer formula

Let M be an oriented, compact, even-dim. Riemannian manifold.

- Let E and F be Hermitian vector bundles on M and let

$$L : C^\infty(M, E) \to C^\infty(M, F)$$

be a "Gauss-Bonnet type operator" (technically called a **Dirac operator**).

Part IV: The Atiyah-Singer formula

Let M be an oriented, compact, even-dim. Riemannian manifold.

- Let E and F be Hermitian vector bundles on M and let

$$L : C^\infty(M, E) \to C^\infty(M, F)$$

be a "Gauss-Bonnet type operator" (technically called a **Dirac operator**).

- (**Atiyah-Singer Index Theorem, 1963**) ind L is Fredholm and the following index formula holds:

$$\boxed{\begin{array}{ccc} \operatorname{ind} L & = & \int_M K_{AS}, \\ \textbf{analytical} & & \textbf{geometrical} \end{array}}$$

where K_{AS} is an **explicitly** defined polynomial in the curvatures of M, E, and F.

Summary of Talk

Question: Can a linear map “detect” topological information?

Answer: Yes, via the index of the operator.

Question: Are the topology and geometry related for a smooth surface? What about the analysis and the geometry?

Answer: Yes, via the Gauss-Bonnet formula; the traditional and index versions.

Question: Can all the above be generalized to higher dimensional manifolds and vector bundles?

Answer: Yes — the Atiyah-Singer index formula.

Questions you may have

- We discussed a “poor man’s” Gauss-Bonnet operator, but a . . .

“poor man’s ____” is a cheaper, simpler version of ____.

So, what is the true Gauss-Bonnet operator?

Questions you may have

- We discussed a "poor man's" Gauss-Bonnet operator, but a . . .

  "poor man's ____" is a cheaper, simpler version of ____.

  So, what is the true Gauss-Bonnet operator?

- What is a Dirac operator?

Questions you may have

- We discussed a “poor man’s” Gauss-Bonnet operator, but a . . .

  “poor man’s ____” is a cheaper, simpler version of ____.

  So, what is the true Gauss-Bonnet operator?

- What is a Dirac operator?

- What is K_{AS}?

Questions you may have

- We discussed a "poor man's" Gauss-Bonnet operator, but a . . .

"poor man's ____" is a cheaper, simpler version of ____.

So, what is the true Gauss-Bonnet operator?

- What is a Dirac operator?
- What is K_{AS}?
- **Answers in next index theory lecture!**