CORRIGENDUM TO "THE TRANSFER IS FUNCTORIAL"

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In the paper [KM], the diagram (13) in the proof of Proposition 6.5 does not strictly commute as claimed. The two branches only agree if we make the additional assumption that the action of H on F is trivial. The authors would like to thank Shachar Carmeli and Bastiaan Cnossen for finding this error and bringing it to their attention.

As a result, the *homotopy* commutativity of (13), and also of the diagram (9), is uncertain. As we explain in Section 3 of [KM], diagram (9) commutes up to homotopy in the fiber bundle case, but it may or may not commute up to homotopy in the general case.

In light of this correction, the paper does not constitute a proof of Theorem A, that the Becker-Gottlieb transfer is functorial for every pair of fibrations $p\colon X \to Y$ and $q\colon Y \to Z$ with finitely dominated fibers. Instead, we get a partial result:

Theorem A. *The transfer $(q \circ p)^!$ agrees in the homotopy category with the composite of transfers $p^! \circ q^!$ in each of the following two cases:*

(1) p is a trivial bundle $Y \times F \to Y$ (with F finitely dominated) and q is arbitrary.

(2) p is arbitrary and q is a (finite sheeted) covering space.

Carmeli and Cnossen also prove that (1) can be extended to the more general case that p is the pullback of a fibration $E \to Z$ along q, see [CCRY]. All together, this gives several cases not already known by the previous result of Dold, that $(q \circ p)^!$ agrees with $p^! \circ q^!$ if both p and q are parametrized ENRs ([D, Thm 8.7]).

We expect in future work to give an argument for functoriality when p is arbitrary but $q\colon Y \to *$ is a projection to a point. Equivalently, that $(q \circ p)^!$ and $p^! \circ q^!$ agree on π_0 for general p and q. We also expect to be able to extend Dold's result to the case where each of p and q is a fiberwise retract of a parametrized ENR.

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However, even after this additional work, the general case will remain open. These special cases provide some evidence that the transfer might be functorial in general. On the other hand, they provide restrictions on the form that a possible counterexample could take.

References

[D] Dold, A.: The fixed point transfer of fibre-preserving maps. *Math. Z.* **148** (1976), 215–244

[CCRY] Carmeli, S., Cnossen, B., Ramzi, M., Yanovski, L.: Characters and transfer maps via categorified traces. *in preparation.*

[KM] Klein, J. and Malkiewich, C.: The transfer is functorial. *Adv. Math.* **338** (2018), 1119–1140

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