

# RESEARCH STATEMENT

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Stable homotopy theory and algebraic  $K$ -theory are deep, rich subjects, that draw a lot of their original motivation from the study of high-dimensional manifolds and fixed-point theory. My research continues this long tradition by finding new relationships between

- trace methods in algebraic  $K$ -theory and fixed-point theory,
- equivariant structures on algebraic  $K$ -theory and equivariant manifolds,
- and most recently, combinatorial forms of algebraic  $K$ -theory and scissors congruence.

To do this research I draw on my experience with equivariant spectra, parametrized spectra,  $K$ -theory, and hands-on geometric techniques for computations, from the previous work [Mal17b, Mal17c, Mal17a, MM19, LM19, LM18, DMP<sup>+</sup>19, MP22, MM20, MM22, CLM<sup>+</sup>20, KM22, Mal22, KMR23, BGM<sup>+</sup>24, GIMM23, Mal23a, MP23, MS24, KLM<sup>+</sup>24]. What is unique about my approach is the extent to which I simplify and streamline the powerful technology of homotopy theory, down to its foundations, as illustrated in [Mal19, Mal23b]. This streamlining has unanticipated but far-reaching consequences. For instance, the calculations in [Mal17c], which set out to simplify earlier computations of TC, turn out to be an important ingredient in the recent disproof of the telescope conjecture [BHLS23].

**0.1. The higher form of Hilbert’s Third Problem.** I would like to start with the project that began in the spring of 2022 and resulted in the paper [Mal22]. It solves the higher version of Hilbert’s Third Problem for each of the one-dimensional geometries. This is a fundamental advance in our understanding of scissors congruence, and the fact that it is even possible has come as a surprise to those in the area.

The scissors congruence problem asks, given two polytopes  $P$  and  $Q$  in a neat geometry (Euclidean, spherical, or hyperbolic), can  $P$  be cut into finitely many pieces and rearranged to form  $Q$ ? The problem is effectively solved for low-dimensional geometries (up to dimension three or four, depending on the geometry). However, it is open in all geometries above dimension four. This problem is often called Hilbert’s Third Problem because Hilbert asked specifically about the case of three-dimensional Euclidean geometry. See e.g. [Dup01, Sah79] for an overview of the progress made on this problem in the second half of the 20th century.

The higher scissors congruence problem, or higher Hilbert’s Third Problem, asks about the *space* formed by taking a point for every polytope, a path for every scissors congruence between polytopes, a 2-simplex for every pair of composable paths, and so on. Group-completing this space gives the scissors congruence  $K$ -theory spectrum  $K(\mathcal{P})$ , first described by Zakharevich [Zak17]. We think of this object as a space whose connected components ( $\pi_0$ ) describe the solution to the classical scissors congruence problem. The higher groups describe something beyond the polytopes up to scissors congruence – they describe *the scissors congruences themselves*.

My contribution to the area is to show that  $K(\mathcal{P})$  is a particularly nice kind of spectrum called a Thom spectrum [Mal22]. It is similar to the landmark result of Madsen and Weiss that resolved the

Mumford Conjecture [MW07] – their result showed that the moduli space of surfaces is described by a Thom spectrum, thereby computing the stable homology of mapping class groups. My result achieves a similar breakthrough but with surfaces replaced with polytopes, and mapping classes replaced by “cut-and-paste” maps between polytopes.

This result allows us to make computations that were not possible before. For instance, we now know that the spectrum is rational for every Euclidean geometry. We also know *all* of its higher  $K$ -groups when the geometry is one-dimensional. This is what solves the higher version of Hilbert’s Third Problem in dimension one.

Several of the intermediate results leading to this are joint work with Bohmann, Gerhardt, Merling, and Zakharevich. In particular, one of the steps involves showing that homotopy orbits commute with a certain form of  $K$ -theory of multicategories [BGM<sup>+</sup>24]. Another gives a new variant of the Solomon-Tits theorem for  $K$ -theory with no isometries – it is equivalent to a wedge of spheres [Mal22]. Both of these intermediate steps are highly interesting on their own.

This breakthrough is already leading to a larger effort to understand and compute the higher  $K$ -groups of scissors congruence. One of the first products of this interdisciplinary collaboration (homotopy theory and neighboring disciplines of mathematics) is my recent paper with Kupers, my student Ezekiel Lemann, Miller and Sroka [KLM<sup>+</sup>24], which shows that the group  $\text{Aut}(P)$  of self-scissors congruences of a polytope  $P$  satisfies a strong form of homological stability. This allows us to compute the homology  $H_*(\text{Aut}(P))$  from the higher  $K$ -groups, just as the homology of the general linear groups  $GL_n(R)$  are related to the algebraic  $K$ -theory of  $R$ . Combining this with [Mal22] gives computations of the homology of the interval exchange transformation group  $IET$ , and a new proof that Thompson’s group  $V$  is acyclic [SW19]. Future work will perform more calculations of scissors congruence in two-dimensional geometries, define the Dehn invariant on the higher scissors congruence groups, prove a higher version of the Dehn-Sydler theorem for spherical geometries (the statement that volume and Dehn invariants separate scissors congruence classes), relate this work to recent developments in group theory via topological full groups [Li22], and expand the celebrated conjectures of Goncharov that relate scissors congruence to the higher algebraic  $K$ -theory of fields [Gon93, CZ19].

The trace methods I developed with my co-authors have also produced many new nonzero classes in the higher  $K$ -groups, and it will take time to sort out all of the new information we have and to present it systematically to the rest of the community. I am excited to report more on these efforts in the coming years.

**0.2. Equivariant manifolds.** The surgery theory program to classify high-dimensional manifolds was one of the greatest successes of topology in the 20th century. When we access the space of diffeomorphisms, algebraic  $K$ -theory plays a critical role, in the form of Waldhausen’s functor  $A(X)$ . The variant of this program for  $G$ -manifolds, when  $G$  is a finite group, is more difficult and the picture is far from complete.

In the paper [MM19], Mona Merling and I develop the equivariant version of Waldhausen’s  $A(X)$ . The desired properties of  $A_G(X)$  were discussed by Goodwillie, Rognes, Waldhausen, and others in the 1990s, but the necessary technology to build equivariant algebraic  $K$ -theory out of Waldhausen categories with appropriate  $G$ -actions has only recently appeared.

Our next two goals are to prove the equivariant analogs of Waldhausen’s celebrated parametrized  $h$ -cobordism theorem and splitting theorem. Together, these theorems give a splitting  $A(X) \simeq \Sigma_+^\infty X \times Wh^{\text{DIFF}}(X)$ , where the second spectrum is a delooping of the stabilized space  $\mathcal{H}^\infty(X)$  of  $h$ -cobordisms on  $X$ .

We achieved the category-theoretic part of this program in the paper [MM22], building on the results from [BO15, BD17]. The next step is the manifold part, which is joint work with Tom Goodwillie, Kiyoshi Igusa, and Mona Merling. The main result is that the stable space of smooth equivariant  $h$ -cobordisms has a splitting

$$(0.2.1) \quad \mathcal{H}_G^\infty(X)^G \simeq \prod_{(H) \leq G} \mathcal{H}^\infty(X_{hWH}^H).$$

This in turn separates into two parts – proving a similar result unstably for *isovariant*  $h$ -cobordisms, and then using functoriality of the  $h$ -cobordism space to pass to the stable space, where isovariant and equivariant  $h$ -cobordisms coincide. The functoriality part of this program is carried out in [GIMM23] – we give a new presentation of the  $h$ -cobordism space as a functor up to coherent homotopy, using several new ideas to streamline earlier treatments, which makes it tractable to generalize the result to manifolds with a  $G$ -action. Even in the non-equivariant case, this is an important development for the literature on the space of  $h$ -cobordisms.

At the level of  $\pi_0$ , the above results recover the smooth equivariant  $s$ -cobordism theorem of Browder-Quinn and Rothenberg [BQ75, Rot78]. Above this level, they allow us to calculate rational stable diffeomorphism groups of  $G$ -manifolds, starting with discs, spheres, and tori.

**0.3. Equivariant Khovanov homology.** In [Kho00] Khovanov introduces knot homology, which categorifies the Jones polynomial, and is a strictly stronger knot invariant. Lipshitz-Sarkar [LS14] further lift this construction from Khovanov homology to a Khovanov spectrum, which is an even finer invariant: there exist smooth knots with the same Khovanov homology but nonequivalent Khovanov spectra [See12]. The Khovanov homology of the infinite twist is defined as the homotopy colimit of the Khovanov homology of the torus knots,

$$\mathrm{Kh}(T_{n,\infty}) = \mathrm{hocolim}(\mathrm{Kh}(T_{n,1}) \rightrightarrows \mathrm{Kh}(T_{n,2}) \rightrightarrows \mathrm{Kh}(T_{n,3}) \dots).$$

This is an example of particular interest, as it can be used to describe a categorified version of the Jones-Wenzl projector in the Temperley-Lieb algebra, which in turn is used to define the colored Jones polynomial and quantum invariants of 3-manifolds [Roz14].

In current work with Bohmann, Klang, Merling, Stoffregen, Willis, and Zhang (about half homotopy theorists and half low dimensional topologists), building on work of Mazel-Gee, Stoffregen and Willis [Wil18, SW24], we show that the “annular” version of the Khovanov spectra  $\mathrm{Kh}^{\mathrm{ann}}(T_{n,\infty})$  are cyclotomic, and moreover, the geometric fixed points of the Khovanov spectra  $\mathrm{Kh}(T_{n,\infty})$  recover the annular Khovanov spectra  $\mathrm{Kh}^{\mathrm{ann}}(T_{n,\infty})$ .

As a necessary step towards this goal, we give a new model of multiplicative equivariant  $K$ -theory [MMO, Yau24, CMNN24] which is both simpler and more general, allowing as input any  $G$ -diagram of symmetric monoidal categories, as defined in [DM16]. It makes critical use of Elmendorf-Mandell  $K$ -theory [EM06] and the Reedy cofibrancy properties studied in detail in my paper [Mal23a]. It is surprisingly simple, which makes its significance extend far beyond Khovanov homology. Indeed, there is a great deal of recent work concerning how to define equivariant  $K$ -theory for equivariant symmetric monoidal categories, including [Mer17, GMMO19, GMMO23, Nar16, CMNN24], and the proposed construction has the potential to make this area broadly accessible, leading to more applications outside of homotopy theory in the near future.

**0.4. Fixed-point theory and its generalizations.** Fixed point theory asks whether the fixed points can be removed from a map  $f: X \rightarrow X$  by a homotopy, if  $X$  is a finite CW complex. The fiberwise variant considers fiber bundles  $E \rightarrow B$  with fiber  $X$ , and fiberwise maps  $f: E \rightarrow E$ . See

e.g. [Dol76]. The primary algebraic obstruction for these problems is the Lefschetz number  $L(f)$ , but it has a refinement called the Reidemeister trace  $R(f)$  that is often a complete obstruction to removing fixed points.

Work of Geoghegan and Nicas in the 1990s on fiberwise Reidemeister traces [GN94a] provided strong hints that in the fiberwise case, topological Hochschild homology (THH) is the correct home for the Reidemeister trace  $R(f)$ . This idea was later realized through work of Klein, Williams, Ponto, Campbell, Lind, and myself [KW07, Pon10, LM19, CP19]. The central concept, from Ponto, is the notion of a “non-commutative” or “bicategorical” trace, generalizing the trace in a symmetric monoidal category that was used by Dold to form the fiberwise Lefschetz number. See Figure 0.4.1. This trace is carried out in a bicategory of parametrized spectra, and my book [Mal19] gives a precise and updated treatment of this category so that future work can proceed on firm footing.

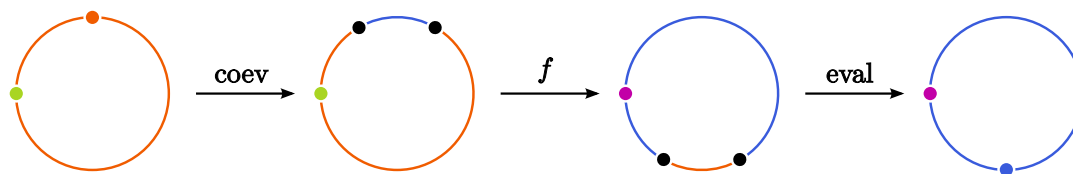


FIGURE 0.4.1. The “non-commutative” or “bicategorical” trace.

In [MP22] Kate Ponto and I consider the problem of removing  $n$ -periodic points from a family of maps  $f: E \rightarrow E$  over  $B$ . We show that a certain invariant called the “Fuller trace” is a generalization of all previously known periodic-point invariants. The Fuller trace is essentially a Reidemeister trace carried out in genuinely equivariant spectra, so this is a surprising application of equivariant stable homotopy theory to a problem in classical topology.

My student Lucas Williams has constructed a counterexample showing that the Fuller trace is in fact strictly stronger, so that equivariant homotopy theory actually produces a better invariant than other methods. This relies on a careful treatment of the equivariant fiberwise version of the Pontryagin-Thom isomorphism. In current work with Ponto and Williams, we show that the Fuller trace is a complete obstruction in families, by generalizing an argument by Jezierski [Jez01] to families of fixed-point problems.

I have also used the Fuller trace to describe an invariant for periodic orbits of flows, in other words periodic solutions to ordinary differential equations. This generalizes a construction by Geoghegan and Nicas [GN94b] and recovers a construction of Michael Crabb. I was able to reinterpret it using trace methods in algebraic  $K$ -theory, showing that it defines a class in topological Frobenius homology (TF). This suggests that there are further connections between trace methods and the dynamics of ordinary differential equations, that have yet to be developed.

**0.5. Generalized characteristic polynomials.** The recent joint project [CLM<sup>+</sup>20] with Campbell, Lind, Ponto, and Zakharevich constructs a very general trace map from  $K$ -theory of endomorphisms to topological restriction homology (TR). For a commutative ring, on  $\pi_0$  this map coincides with the characteristic polynomial from [Alm74]. In general, each endomorphism of an  $R$ -module goes to a class in TR that is described as a Fuller trace, just like in the work on periodic-point theory above. In other words:

*The Fuller trace is a topological, non-commutative generalization of the characteristic polynomial.*

This result fits the previous work with Ponto on periodic points into a larger  $K$ -theoretic context. It has other upshots as well. One is that, as in [LM19], these Fuller traces lie underneath wrong-way maps in algebraic  $K$ -theory.

**0.6. Transfers in algebraic  $K$ -theory and Hochschild homology.** John Lind and I began importing the Reidemeister trace from fixed-point theory into algebraic  $K$ -theory in [LM19]. It gives us a formula for “wrong-way” maps on THH lying underneath  $K$ -theory. This is a powerful technique. In that paper, it allowed us to prove that the Becker-Gottlieb transfer of [BG75] is a retract of the  $A$ -theory transfer of [Wil00], generalizing earlier results from [DJ12] and elsewhere. It also allowed us to do homology calculations of the transfer for the Hopf bundle.

With a tractable amount of additional work, this technique should allow us to compute  $K$ -theory transfer maps rationally. Through the connection between  $K$ -theory and differential topology, see e.g. [DWW03], this can be used to identify exotic families of smooth structures on manifold bundles. My student Shuchen Mu has been doing calculations of these that start with the case of spherical fibrations. Another upshot of the planned work in this area is additional calculations of the fiberwise Reidemeister trace over a base of dimension greater than one, and a new technique that uses the main result of [LM19] to compute the involution on the free loop space  $LX$  rationally. This latter calculation can be used to get rational stable diffeomorphism groups of various manifolds, see e.g. [WW14, BFJ17].

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