Using the facts that an extract of a set for any kind of extraction (direct extracts, the direct extracts of the preceding one, etc.), the extracts of a line determine the same function as the line, it follows [using the notion of partial table] that by replacing lines in a weak table (for a permutation) by a set of extracts, we can always arrange from two weak tables in 'nice' form (right sequence of A-line ending in \(4\)) to obtain the composition; hence we have a way of representing the elements of \(G\) by finite (modulo \(A\)) artifacts — and multiplying them effectively. To check whether such an artifact represents the identity is easy: if \(G\) has a divisible W.P. — if \(A\) does.

Start: Suppose that \(A' = A\) (\(A'\) is commutator subgroup of \(A\)).

To show that \(G\) is simple: say \(a \in A\), and \(I\) in the factor group, but \(a \neq I\). Then for some finite binary sequence \(\sigma, \tau\), we have \(a \neq I\) — in fact, \(\sigma\) and \(\tau\) determine disjoint intervals — and \(\sigma \rightarrow \tau\) is an extract of a line of \(A\). Hence, if \(X \in E\), and \(X_{\sigma'}, X_{\tau'}\) are the 'similar' actions \(\frac{X}{\sigma}\) on \(\sigma\) and \(\tau\), we have \(a^{-1} X_{\sigma'} a = X_{\tau'}\).

Hence, \(a' X_{\sigma'} a = X_{\tau'}\); \(\sigma X_0 \sim X_{\tau'}\); As \(X_0 \neq X_{\tau'}\) (since \(\sigma \neq \tau\) and \(X \neq 1\)), we obtain a relation in \(E\). As \(E\) is simple, this means \(X \sim I\) for all \(X \in E\) in the factor group. The easily collapses to whole group \((\forall g, h \in A^\times, \exists p \in X, (p, p), A I = I \Rightarrow h g \sim h g \& i\). \(A^\times\) collapses to Abelian. But \(A^\times = A\) & \(A' = A\), so \(A^\times\) collapses to \(I\).

(Note: apply the 2nd Postcard to get from \(A^\times = A\), if \(A^\times\) is divisible.)
Let $A$, with a solvable word problem, be a group of permutations of the set $T$ of positive integers and such that for $g, h \in T$ and $g \in A$, it is decidable whether or not $g(h) = k$. (Convention: $(g^h)(j) = h(g(j)))$.

Now let $\mathcal{T}$ be the set of all finite sets $B$ of ordered pairs $\langle m, g \rangle$, $m \in T$, $g \in A$, such that for all $m \in T$, $g, h \in A$, $\langle m, g \rangle, \langle m, h \rangle \in B \implies g = h$.

And let $\mathcal{A}(B)$, for $B \in \mathcal{T}$, be $\{ \langle m+1, g \rangle : \langle m, g \rangle \in B \}$. Then $\mathcal{A}(B) \in \mathcal{T}$.

Let us call an ordered pair $\langle B, m \rangle$ whose $B \in \mathcal{T}$ and $m = 0$ on $m = T$ acceptable if for every $\langle m, g \rangle \in B$ we have, for all $x \geq m$, $g(x) \equiv m \geq m$.

For $m \in T$, and $B, C \in \mathcal{T}$, let $B \circ C = \{ \langle m, g \rangle : \langle m, g \rangle \in B \in C \in A \land \langle m, h \rangle \in B \land \langle m, g \rangle \in C \land \exists k \in T, v \in A (\langle m, k \rangle \in v(B)) \}$. Then $B \circ C \in \mathcal{T}$.

Let $B[g]$ be the $\{ \langle 1, g \rangle \}$, for $g \in A$. Then $B[g] \in \mathcal{T}$.

Regular lines are of the form: $\sigma \rightarrow \tau$, where $\sigma$ and $\tau$ are finite binary sequences; $\sigma$ is the left sequence $\tau$ the right sequence of the line.

A line is a line of the form: $\sigma \cdot \tau, m$, where $\sigma$ and $\tau$ are finite binary sequences, $m \in T$, and $B \in \mathcal{T}$; again, $\sigma$ is the left sequence of $\tau$ the right sequence of the line.

For $\sigma$ a finite binary sequence, let $[\sigma] = \{ B \in \mathcal{T} : \sigma \subseteq B \}$, where $\subseteq$ is equal to the set of all binary sequences of length $\sigma$ which are met eventually either $0$ or $1$.

The composition of lines: We define:

1. $(\sigma \rightarrow \tau) \cdot (\tau \rightarrow \rho) = (\sigma \rightarrow \rho)
2. (\sigma \rightarrow \tau) \cdot (\tau \rightarrow \rho, m) = (\sigma \rightarrow \rho, m)
3. (\sigma \rightarrow \tau, m) \cdot (\tau \rightarrow \rho) = (\sigma \rightarrow \rho, m)
4. (\sigma \rightarrow \tau, m) \cdot (\tau \rightarrow \rho, m + \omega) = (\sigma \rightarrow \rho, m + \omega)$.
weak tables

A **collection** of lines, all of whose right sequences are distinct such that the set $\mathcal{P}$ of left sequences of the lines is such that $\mathcal{P} = \{\alpha \in \mathbb{N} : \alpha \leq n\}$ forms a partition of $\mathcal{P}$, and such that if $t \rightarrow t, m$ belong to $\mathcal{P}$, then $<t, m>$ is acceptable.

The direct extracts of a regular line $t \rightarrow t$ are the lines $t \rightarrow t$ and $t \rightarrow t'$.

The direct extracts of an A-line $\sigma \rightarrow t, m$, where $<t, m>$ is acceptable, are the lines (where $m^* = \max \{m : \exists t, m \in \mathcal{P}, \alpha \in \mathbb{N} \}$ )

$$\sigma^1 \rightarrow t, m+1$$

$$\sigma^m \rightarrow t^m, m+2$$

and for $0 < m < m^*$:

$$\sigma^{m^*} \rightarrow t^{m^*}, m^* + 1$$

provided that $<m, g> \in \mathcal{P}$, $<m, h> \notin \mathcal{P}$.

An extract of a line is a line obtained as the last in a series of direct extracts, starting from the given line.

**Lemma A:** If $\mathcal{P}$ is a finite binary sequence terminating in $t$, and $\sigma$ is an $A$-line extending $\sigma'$, then if $\sigma \rightarrow t, m$ is an A-line and $<t, m>$ is acceptable, there is some line which is an extract of the line whose left sequence is $\sigma'$; moreover, there is only one such line.

Given two weak tables such that $t$ is the right sequence of the terminal row, $\mathcal{P}_1$ extends to an A-line, and an extension of a left sequence of the second, $\mathcal{P}_2$, we define the composition $\mathcal{P}_1 \circ \mathcal{P}_2$ as follows: We replace each line of $\mathcal{P}_2$ by as follows.

The following line in the line $\sigma \rightarrow t$ or $\sigma \rightarrow t, m$ is replaced, where $t'$ is the unique (by the partition property of the left sequences of $\mathcal{P}$) left sequence containing $t$ occurs in a $\mathcal{P}_2$ line (denoted $t'$). The line $\mathcal{P}_1 \circ \mathcal{P}_2$ is obtained from $\mathcal{P}_1$ by the following operation: The $\mathcal{P}_1$ line with $t' \rightarrow t^* \rightarrow \mathcal{P}_2$ is replaced by the composition of the $\mathcal{P}_1$ line with $t' \rightarrow \mathcal{P}_2$, $\mathcal{P}_2$ being the binary sequence such that $t = t'$.

If instead $t' \rightarrow t^* \rightarrow \mathcal{P}_2$ is the unique line of $\mathcal{P}_1$ with $t'$ as its left sequence, the $\mathcal{P}_1$ line is replaced by the composition of the $\mathcal{P}_1$ line with the unique line, guaranteed by Lemma A, which is an extract of $t' \rightarrow t^* \rightarrow \mathcal{P}_2$ and has $t'$ as its left sequence (as $t'$ extends $t^*$ and $t$ ends in $t'$).
Every \( \beta \in \mathcal{K} \) has the form \( \langle 1, g \rangle^{m} \langle 0 \rangle^{n} \langle 1 \rangle^{\ell} \) for some \( \ell, g \in \mathbb{Z} \) and \( m, n \in \mathbb{N} \). Let \( \mathcal{J} \) be its index.

The function defined by the line \( \sigma \rightarrow \tau \) is that function \( \chi \) with domain \( [0] \) such that \( \chi(\sigma^{\beta}) = \tau^{\beta} \) for all \( \beta \in \mathcal{K} \).

The function defined by the line \( \sigma \rightarrow \tau \), where \( \langle \beta, m \rangle \) is acceptable, is that function \( \chi \) with domain \( [0] \) such that:

1. If \( \beta \) has parity \(-m\), \( \sigma(\beta) = 0 \), and \( \langle m, g \rangle \in \mathcal{J} \):
   \[ \chi(\sigma^{\beta}) = \tau^{\langle 1 \rangle^{g(\ell+m)-m} \langle 0 \rangle^{g(\ell)} \langle 0 \rangle \langle 1 \rangle^{\ell} \beta} \text{.} \]
   Here \( \beta = \langle 1 \rangle^{g-m} \langle 0 \rangle^{k} \beta' \) and \( k > m \), hence \( g(\ell) + m > m \), \( \langle 0 \rangle \beta' \) being acceptable.

2. If \( \beta \) does not have parity \(-m\) for any \( \langle m, g \rangle \in \mathcal{J} \):
   \[ \chi(\sigma^{\beta}) = \tau^{\langle 0 \rangle^{m} \langle 1 \rangle^{\ell} \beta} \text{.} \]

The function defined by a weak table \( \Pi \) is that function \( \chi \) with domain \( \mathcal{K} \) (as the left sequences of the lines of \( \Pi \) will intervals forming a partition of \( \mathcal{K} \)), which is the union of the functions defined by the lines.

A partial table is a set of regular lines and \( \lambda \)-lines \( \sigma \rightarrow \tau, m \) (such that \( \langle 0, m \rangle \) is acceptable) such that all the left sequences are distinct and incompatible; the function defined by the partial table is the union of the line function.

For \( g \in A \) let \( g^{*} \in \mathcal{K} \) be the function defined as follows:

\[ g^{*}(\langle 1 \rangle^{g} \langle 0 \rangle^{m} \langle 0 \rangle^{n} \langle 1 \rangle^{\ell} \beta) = \langle 1 \rangle^{g(m)} \langle 0 \rangle^{g(n)} \langle 0 \rangle^{g(\ell)} \langle 0 \rangle \langle 1 \rangle^{\ell} \beta \text{ for } m \in \mathbb{Z}, \beta \in \mathcal{K}. \]

\[ g^{*}(\beta) = \beta \text{ otherwise.} \]

Then \( g^{*} \) is a permutation since \( g \) is.

Follows \( A^{*} = \langle g^{*} ; g \in A \rangle \), and \( C' \) is the group of permutations of \( \mathcal{K} \) which are defined by weak tables with only regular lines (thus, to give permutations, it is necessary and sufficient that the right sequences also form yield intervals which partition \( \mathcal{K} \), the into right sequences all being distinct). Let \( G \) be the group generated by \( A^{*} \cup C' \) (\( C' \) is finitely generated).

Let \( G_{0} \) be the group of all permutations of \( \mathcal{K} \) such that, for every \( \beta \in \mathcal{K} \), there is some finite sequence \( \sigma \) such that \( \beta = \sigma^{\beta} \) and \( \pi(\sigma^{\beta}) = \tau^{\beta} \) for some finite sequence \( \tau \), for all \( \beta \in \mathcal{K} \).