Math 222 Calculus 2 Spring 2000 Practice Exam 2 Feingold

Evaluate: (1) (8 Points) $\displaystyle\int_1^9 \frac{1}{\sqrt[3]{x-9}}\,dx$ (2) (8 Points) $\displaystyle\int_1^\infty \frac{\ln(x)}{x^3}\,dx$

(3) (7 Points) Find the area of the surface of revolution made by rotating the curve $x = \sqrt{2y - y^2}$ for $0 \le y \le 1$ around the y-axis.

(4) (7 Points) Write down the integral for the area of the surface of revolution made by rotating the curve $y = e^{-x}$ for $0 \le x \le 1$ around the x-axis, but DO NOT TRY TO COMPUTE THE INTEGRAL.

(5) (7 Points) Find the length of the curve $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$ for $1 \le x \le 2$.

(6) (8 Points) Use the Comparison Theorem to determine whether the following improper integral converges or diverges. DO NOT COMPUTE THE EXACT VALUE OF THE INTEGRAL, but show all work needed for the Comparison Theorem.

$$\int_2^\infty \frac{1}{\sqrt{x^2+2}}\,dx$$

(7) (8 Points) Solve $\dfrac{dy}{dx} = \dfrac{1+x}{xy}$ with initial conditions $x > 0$ and $y(1) = -4$.

(8) (10 Points)

(a) Write out (BUT DO NOT ADD UP) the Simpson's Rule sum with $n = 10$ which approximates $\displaystyle\int_1^{11} \ln(x)\,dx$.

(b) Find the least n such that the Simpson's Rule error

$$|E_S| \le \frac{K(b-a)^5}{180n^4} \text{ for } |f^{(4)}(x)| \le K$$

of the approximation of the integral in part (a) is guaranteed less than .0001.

(9) (10 Points) Determine whether each sequence $\{a_n\}_{n=1}^\infty$ converges or diverges, and if it converges, then find its limit.

(a) $a_n = \sqrt{n+3}(\sqrt{n+5} - \sqrt{n})$ (b) $a_n = \dfrac{\ln(2n^2+1)}{\ln(3n^2+2)}$

(10) (15 Points) Determine whether each series converges or diverges. Explain the reasons for your answer. If the series converges, find its sum.

(a) $\displaystyle\sum_{n=1}^\infty \frac{n^2}{8(n+1)(n+2)}$ (b) $\displaystyle\sum_{n=2}^\infty \frac{4}{n(n-1)}$ (c) $\displaystyle\sum_{n=1}^\infty \frac{e^{n+1}}{\pi^{n-1}}$

(11) (12 Points)

(a) Find the half-life of a radioactive substance if a 100 gram sample is reduced to 25 grams in 150 years.

(b) If $f'(t) = 9(f(t) - 5)$ and $f(0) = 8$ then find $f(t)$.