Math 222 Calculus 2 Fall 2001 Practice Final Exam

(1) (15 Points) Parametric equations for a curve are $x = t^2 + e^t$ and $y = t - \sin(t)$ for $0 \leq t \leq 1$.

(a) Setup the integral for the length of that curve, but DO NOT EVALUATE OR SIMPLIFY THAT INTEGRAL.

(b) Set up the integral for the surface area obtained by rotating that curve around the y-axis, but DO NOT EVALUATE OR SIMPLIFY THAT INTEGRAL.

(c) Find the equation of the tangent line to that curve at $t = 0$.

(2) (10 Points) Write the Taylor polynomial $T_3(x)$ of degree 3 for the function $f(x) = x^6$ centered at $a = -2$.

(3) (10 Points) Find an explicit solution $y = f(x)$ for the differential equation $\dfrac{dy}{dx} = \dfrac{(x^2)(y^2+1)}{2y}$ with initial condition that $y = 2$ when $x = 0$.

(4) (10 Points) If possible, write each of the following series as a reduced fraction.

(a) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{-1}{4}\right)^n$

(b) 0.2272727...

(5) (15 Points) Find the interval of convergence for each of the following power series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\ (2x+5)^n}{n\ 4^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3\ x^n}{n!}$

(6) (15 Points) Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent. Explain all details of the tests you are applying.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n\pi}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\ \frac{10 - 2n^2}{10^{25} + 7n^2}$

(7) (20 Points) The polar equation $r = 1 + 2\cos^2(\theta)$ determines a curve for $0 \leq \theta \leq 2\pi$.

(a) Sketch that curve on the graph paper below.

(b) Setup an integral for the area enclosed by that curve just for $0 \leq \theta \leq \pi/2$, but DO NOT EVALUATE OR SIMPLIFY IT.

(8) (30 Points) Find each of the following, if possible.

(a) $\displaystyle\int_{\sqrt{3}}^{\infty} \frac{1}{1+x^2}\ dx$

(b) $\displaystyle\lim_{x\to 0} \frac{x\sin(x)}{1-\cos(x)}$

(c) $\displaystyle\lim_{x\to 0} \frac{x^4\ e^x}{\cos(x)-1+x^2/2}$

(9) (30 Points) Evaluate the following integrals.

(a) $\displaystyle\int \sin^3(x)\ \sqrt{\cos(x)}\ dx$

(b) $\displaystyle\int (4x^3+2x)\ln(x^2+1)\ dx$

(c) Find $\displaystyle\int x^3\ \sqrt{1-9x^2}\ dx$

(10) (10 Points) Suppose the alternating series $\displaystyle\sum_{k=2}^{\infty} \frac{(-1)^k}{3+\log_{10}(k)}$ converges to S and write the N^{th} partial sum as $S_N = \displaystyle\sum_{k=2}^{N} \frac{(-1)^k}{3+\log_{10}(k)}$. What is the smallest value of N such that you can be sure $|S - S_N| < \frac{1}{9}$?

(11) (15 Points)

(a) Use the integral test to show that the positive series $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\ (\ln\ k)^2}$ converges.

(b) If it converges to S and S_N denotes the N^{th} partial sum, use the integral remainder estimate to give an upper bound on $S - S_N$. Find the least N such that $S - S_N \leq \frac{1}{5}$.

(12) (20 Points) Write each of the following functions as a power series, and give its radius of convergence. Write your answer using summation notation or write at least the first four nonzero terms of the infinite series.

(a) $\sin(x)$

(b) $e^{(x^3)}$

(c) $\ln(1-3x)$

(d) $\arctan(x^2)$