Preparation to Final Exam

Let $\triangle x = \frac{b-a}{n}$, $x_0 = a$, $x_n = b$, $x_i - x_{i-1} = \triangle x$, and $\bar{x}_i = (x_{i-1} + x_i)/2$, $i = 1, ..., n$.

1.1. By the Midpoint Rule, $\int_a^b f(x)dx \approx$ Hh $\triangle x \sum_{i=1}^n f(\bar{x}_i)$

Error bound $|E_M| \le \frac{K(b-a)^3}{24n^2}$, where $|f''(x)| \le K$ for x in $[a,b]$.

1.2 By Simpson's Rule, when n is even,

$\int_a^b f(x)dx \approx \frac{\triangle x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$

Error bound $|E_S| \le \frac{M(b-a)^5}{180n^4}$, where $|f^{(4)}(x)| \le M$ for x in $[a,b]$.

1.3. $\int_1^\infty \frac{1}{x^p} dx$ is convergent if and only if $p > 1$

1.4. The arc length of the curve $y = f(x)$, $a \le x \le b$ is $\int_a^b \sqrt{1 + (f'(x))^2} dx$

1.5. The area of a surface of revolution if rotating the curve $y = f(x)$, $a \le x \le b$ about $x-$axis is $S = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2} dx$.

The area of a surface of revolution if rotating this curve about $y-$axis is $S = 2\pi \int_a^b x\sqrt{1 + (f'(x))^2} dx$.

1.6. Comparison Theorem: Suppose f and g are continuous and nonnegative,

a. If $f \le g$ for $x \ge a$ and $\int_a^\infty g(x)dx$ converges then $\int_a^\infty f(x)dx$ converges;

b. If $f \ge g$ for $x \ge a$ and $\int_a^\infty g(x)dx$ diverges then $\int_a^\infty f(x)dx$ diverges.

1.7. The form of the partial fraction decomposition of $\frac{1}{x^3(x^2+x+1)^2(x-2)}$ (**no need to solve for the coefficients**) is $\frac{A_1}{x} + \frac{A_2}{x^2} + \frac{A_3}{x^3} + \frac{a_1x+b_1}{x^2+x+1} + \frac{a_2x+b_2}{(x^2+x+1)^2} + \frac{c}{x-2}$

1.8. The solution to the differential equation $y' = ky$ is $y = y(0)e^{kt}$

1.9. The solution to the differential equation $y' = f(y)g(x)$ is $\int \frac{1}{f(y)} dy = \int g(x)dx$

1.10. L'Hospital's Rule. If $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ exists and either $\lim_{x\to a} f(x) = 0 = \lim_{x\to a} g(x)$ or $\lim_{x\to a} f(x) = \pm\infty = \lim_{x\to a} g(x)$ then $\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$.

1.11. $\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$

1.12. If f is continuous, then $\frac{d}{dx}\int_0^x f(t)dt = f(x)$

1.13. $g(g^{-1}(x)) = x$ and $(g^{-1}(x))' = \frac{1}{g'(g^{-1}(x))}$

1.14. $[f(u(x))]' = f'(u(x))u'(x)$

1.15. $\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$ 1.16. $\frac{d}{dx}\cot^{-1} x = \frac{-1}{1+x^2}$

1.17. $\frac{d}{dx}\sin^{-1}(g(x)) = \frac{g'(x)}{\sqrt{1-g^2(x)}}$ 1.18. $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$

1.19. $\frac{d}{dx}\sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$ 1.20. $\frac{d}{dx}\cot x = -\csc^2 x$

1.21. $\int \sin x dx = -\cos x + c$ 1.22. $\int \cos x dx = \sin x + c$

1.23. $\int \sec x dx = \ln|\sec x + \tan x| + c$ 1.24. $\int \csc x dx = \ln|\csc x - \cot x| + c$

1.25. $\int \tan x dx = \ln|\sec x| + c$

1.26. $\int \sec^2 x dx = \tan x + c$ 1.27. $\int x^{-1} dx = \ln|x| + c$

1.28. $\int a^x dx = (a^x/\ln a) + c$ 1.29. If $n \ne -1$, then $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

1.30. $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$

1.31. $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$ and $\sin^2 A = \frac{1}{2}[1 - \cos(2A)]$

1.32. $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$ and $\cos^2 A = \frac{1}{2}[1 + \cos(2A)]$

1.33. Strategy for evaluating $\int \sin^m x \cos^n x dx$ if n is odd: use the substitution $u = \sin x$ and identity $\cos^2 x = 1 - \sin^2 x$

1.34. Strategy for evaluating $\int \sin^m x \cos^n x dx$ if m is odd: use the substitution $u = \cos x$ and identity $\sin^2 x = 1 - \cos^2 x$

1.35 Strategy for evaluating $\int \sin^m x \cos^n x dx$ if n and m are even: use identities $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ and $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

1.36. Strategy for evaluating $\int \tan^m x \sec^n x dx$ if m is odd: use the substitution $u = \sec x$

1.37. Strategy for evaluating $\int \tan^m x \sec^n x dx$ if n is even: use the substitution $u = \tan x$

1.38. Table of Trigonometric Substitution in integration.

$$\begin{pmatrix} \textit{expression} & \textit{substitution} & \textit{identity} \\ \sqrt{a^2 - x^2} & x = a\sin\theta & \cos^2\theta = 1 - \sin^2\theta \\ \sqrt{a^2 + x^2} & x = a\tan\theta & \sec^2\theta = 1 + \tan^2\theta \\ \sqrt{x^2 - a^2} & x = a\sec\theta & \tan^2\theta = \sec^2\theta - 1 \end{pmatrix}$$

Part 2.

2.1 The sequence $\{\frac{1}{n^p}\}_{n\geq 1}$ converges iff $p \quad \geq 0 \quad$.

2.2 The p-series $\sum \frac{1}{n^p}$ converges iff $p \quad > 1 \quad$.

2.3 The geometric series $\sum_{n=0}^{\infty} ax^n$ converges to $\frac{a}{1-x}$ iff $|x| < 1$.

2.4 The Integral Test. If (1) f is continuous and decreasing and (2) $a_n = f(\ \text{n} \ ) > 0$, $n \geq k$, then $\sum_{n=1}^{\infty} a_n$ converges iff $\int_k^{\infty} f(x)dx$ converges.

2.5 Divergence Test. If we do not have $\lim_{n\to\infty} a_n = \quad 0 \quad$, then $\sum a_n$ diverges.

2.6 Comparison Test. Suppose $0 \leq a_n \leq b_n$. If $\sum \ b_n$ converges, then so does $\sum \ a_n$. If $\sum \ a_n$ diverges, then so does $\sum \ b_n$.

2.7 Limit Comparison Test. If $a_n, b_n \geq 0$, c is finite, and $\lim_{n\to\infty} a_n/b_n = c > \quad 0 \quad$, then $\sum_n a_n$ converges iff $\sum_n b_n$ does.

2.8 The Alternating Series Test. Suppose b_n is decreasing and $\lim_{n\to\infty} b_n = \ 0 \ $. Then $\sum_n (-1)^n b_n$ converges.

2.9 The Ratio Test. $\sum a_n$ converges absolutely if $\lim_{n\to\infty} \ |\frac{a_{n+1}}{a_n}| < 1$, diverges if $\lim_{n\to\infty} \ |\frac{a_{n+1}}{a_n}| > 1$.

2.10 The Root Test. $\sum a_n$ converges absolutely if $\lim_{n\to\infty} \ \sqrt[n]{|a_n|} < 1$, diverges if $\lim_{n\to\infty} \ \sqrt[n]{|a_n|} > 1$.

2.11 The $k-$th Taylor polynomial of f at a is $T_k(x) = \sum_{n=0}^{k} \ \frac{f^{(n)}(a)}{n!}(x-a)^n$.

2.12 The Taylor series of f at a is $\sum_{n=0}^{\infty} \ \frac{f^{(n)}(a)}{n!}(x-a)^n$.

2.13 Taylor's inequality: $|R_n(x)| \leq \frac{K|x-a|^{n+1}}{(n+1)!}$ for $|x - a| \leq d$, if $|f^{(n+1)}(x)| \ \leq K$ for $|x - a| \leq d$.

2.14 $\frac{1}{1-x} = \ \sum_{n=0}^{\infty} x^n$ for x in (-1,1) .

2.15 $e^x = \ \sum_{n=0}^{\infty} \frac{x^n}{n!}$ for x in $(-\infty, \infty)$.

2.16 $\sin x = \ \sum_{n=0}^{\infty}(-1)^n x^{2n+1}/(2n+1)!$ for x in $(-\infty, \infty)$.

2.17 $\cos x = \ \sum_{n=0}^{\infty}(-1)^n x^{2n}/(2n)!$ for x in $(-\infty, \infty)$.

2.18 $\tan^{-1} x = \sum_{n=0}^{\infty}(-1)^n x^{2n+1}/(2n+1)$ for x in $[-1,1]$.

2.19 $\ln(1-x) = -\sum_{n=1}^{\infty} x^n/n$ for x in $[-1,1)$.

2.20 $(\sum_{n=0}^{\infty} a_n x^n)' = \sum_{n=0}^{\infty}(n+1)a_{n+1}x^n$.

2.21 $\int \sum_{n=0}^{\infty} a_n x^n dx = C + \sum_{n=1}^{\infty} a_{n-1}x^n/n$.

2.22 The binomial series of $(1+x)^r = \sum_{n=0}^{\infty} \binom{r}{n} x^n$, where $\binom{r}{0} = 1$ and $\binom{r}{n} = r(r-1)...(r-n+1)/n!$.

The following facts are about a curve C given by a parametric equations C: $x = f(t)$ and $y = g(t)$, $\alpha \leq t \leq \beta$:

2.23 $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ and $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(\frac{dy}{dx})}{\frac{dx}{dt}}$.

2.24 The area between the curve C and the x-axis is $A = \int_{\alpha}^{\beta} g(t)f'(t)dt$

2.25 The arc length of the curve C is $L = \int_{\alpha}^{\beta} \sqrt{[f'(t)]^2 + [g'(t)]^2} \quad dt$

2.26 The surface area is $S = \int_{\alpha}^{\beta} 2\pi g(t)\sqrt{[f'(t)]^2 + [g'(t)]^2}dt$ if C rotates about x-axis.

2.27 The surface area is $S = \int_{\alpha}^{\beta} 2\pi f(t)\sqrt{[f'(t)]^2 + [g'(t)]^2}dt$ if C rotates about y-axis.

The following facts are about a curve C given by Polar coordinates C: $r = f(\theta)$, $a \leq \theta \leq b$.

2.28 $\frac{dy}{dx} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$ (**in terms of θ only**)

2.29 The area of the polar region bounded by the curve C and the rays $\theta = a$ and $\theta = b$ is $A = \int_a^b \frac{1}{2} f(\theta)^2 d\theta$.

2.30 The arc length of the curve C is $L = \int_a^b \sqrt{f(\theta)^2 + (f'(\theta))^2}d\theta$