Exam I, Math 222

October 10, 2002

Problem 1. a) Let $f(x) = x + e^x$.

i) Prove that f has an inverse function g and find the domain of g (**5 points**).

ii) Compute $g(1)$ and $g'(1)$ (**5 points**).

iii) What is $f(g(f(g(5))))$? (**4 points**)

b) (**5 points**) Find the inverse function of $f(x) = e^{\tan x}$, $x \in (-\pi/2, \pi/2)$.

Problem 2. a) (**5 points**) Solve the equation

$$\log_3(x+1) + \log_3(5-x) = 2$$

b) (**5 points**) Find the derivative of the function

$$f(x) = \sqrt{x} e^{\sin x} (x^2+1)^{12}$$

Problem 3. Compute the following limits (**5 points each**):

$$\text{a) } \lim_{x\to\infty} (e^x + x)^{1/(x+1)} \qquad \text{b) } \lim_{x\to 0} \frac{x^2 - x}{\cos x} \qquad \text{c) } \lim_{x\to 0} \frac{\sin x - x}{x^3}$$

Problem 4. Compute the following integrals (**5 points each**):

$$\text{a) } \int x^2 \ln x dx \qquad \text{b) } \int_0^1 x^2\sqrt{x^3+1}dx \qquad \text{c) } \int \sin 2x \cos 3x dx$$

$$\text{d) } \int \frac{dx}{\sqrt{(1+x^2)^5}} \qquad \text{e) } \int \frac{dx}{x^2 - 6x + 13} \qquad \text{f) } \int \frac{x^5 - 4}{x^3 - x^2 - x - 2} dx$$

Problem 5. (**6 points**) Write out (**BUT DO NOT ADD UP**) the Midpoint's Rule sum with $n = 8$ for $\int_0^4 e^{2x} dx$. How large should be n so that the error of the Midpoint's Rule approximation of this integral is less than 10^{-4}?