Exam II, Math 222

Problem 1. a) Is the integral $\int_1^\infty \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}}dx$ convergent or divergent? Justify your answer (**5 points**).

b) Evaluate each of the following integrals or show that it diverges (**5 points each**).

$$\text{(i)} \int_1^\infty \frac{\ln x}{x^2}dx \qquad\qquad \text{(ii)} \int_{-2}^3 \frac{dx}{x^4}$$

Problem 2. a) (**5 points**) Find the length of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$, $1 \le x \le 2$.

b) (**5 points**) Find the area of the surface obtained by rotating the curve $x^2 + y^2 = 1$, $0 \le x \le 1$ about the $x-$axis.

c) (**5 points**) Find the area of the surface obtained by rotating the curve $y = 1 - x^2$, $0 \le x \le 1$ about the $y-$axis. Discuss the two methods you can use here.

Problem 3. a) (**5 points**) Use Euler's method with step size 1 to estimate $y(3)$, where $y(x)$ is the solution of the differential equation $y' = x + 2y$, $y(0) = 1$.

b) (**5 points**) Solve the differential equation $y' = 2 + 2x^2 + y + x^2y$.

c) (**5 points**) A bacteria culture starts with 1000 bacteria and the growth rate is proportional to the number of bacteria. After 2 hours the population is 9000.

i) find an expression for the number of bacteria after t hours;

ii) find the number of bacteria after 3 hours;

iii) how long does it take for the number of bacteria to double?

Problem 4. Compute the following limits (**5 points each**):

$$\text{a)} \lim_{n\to\infty} (1 + \frac{2}{n})^{3n-1} \qquad \text{b)} \lim_{n\to\infty} \sqrt[n]{2^n n^2 + n} \qquad \text{c)} \lim_{n\to\infty} \frac{2^n + 3}{3^n + 2}$$

Problem 5. a) Determine whether each series converges or diverges. Explain the reason for your answer. If the series converges, find the sum (**5 points each**).

$$\text{a)} \sum_{n=0}^\infty \frac{3^{n+1}}{\pi^n} \qquad \text{b)} \sum_{n=0}^\infty \frac{n^2 - 1}{n^2 + 3n - 1} \qquad \text{c)} \sum_{n=2}^\infty \frac{1}{n^2 - 1}$$

b) (**5 points**) Find all x for which the series $\sum_{n=0}^\infty 2^{n+1}x^n$ converges.

Problem 6. (Optional, you may earn 12 extra points) The sequence (a_n) is defined by $a_1 = 1$, $a_{n+1} = \sqrt[3]{3a_n + 2}$. Prove that this sequence converges and find its limit.