Exam III, Math 222

Problem 1. a) Let f be a function which has derivatives of all orders. Define the $n-$th Taylor polynomial of f centered at a and the Taylor series of f centered at a. State the definition of the $n-$th remainder and the Taylor's inequality for the remainder. (**6 points**)

b) Find the Taylor series for $f(x) = \sin x$ centered at $\pi/2$. Use Taylor's inequality to prove that the Taylor series converges to $f(x)$ for all x. (**5 points each**)

c) Find the Taylor series expansion centered at 0 for $f(x) = x\ln(1-x^2)$. Use it to compute $f^{(9)}(0)$. (**5 points each**)

Problem 2. a) State the ratio and root tests. (**6 points**)

b) Determine whether the following series converges absolutely, converges conditionally or diverges (**5 points each**)

$$\text{a)} \sum_{n=0}^{\infty} \frac{(-3)^{1+3n}}{n^n} \qquad \text{b)} \sum_{n=0}^{\infty} \frac{n!}{e^n}.$$

Problem 3. a) State the integral test and the alternating series test (**6 points**)

b) Determine whether the following series converges absolutely, converges conditionally or diverges (**5 points each**)

$$\text{a)} \sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n} \qquad \text{b)} \sum_{n=0}^{\infty} \frac{n}{n^2+1}.$$

Problem 4. Compute the following limits (**5 points each**):

$$\text{a)} \lim_{n\to\infty} (1 - \frac{2}{n})^{3n+1} \qquad \text{b)} \lim_{n\to\infty} \sqrt[n]{2^n + 3^n} \qquad \text{c)} \lim_{n\to\infty} \frac{n2^n + 1}{3^n + 2}$$

Problem 5. A curve C is given by parametric equations $x = \sin t$, $y = \cos^3 t$, $t \in [0, 2\pi]$.

a) Compute dy/dx and d^2y/dx^2 (i.e. the first and second derivatives of y as a function of x). Use it to sketch the curve (find where the curve is concave up or concave down) (**5 points**).

b) Find all $t \in [0, 2\pi]$ for which the tangent to the curve C is horizontal (**5 points**).

c) Find the area of the region bounded by the curve C (**6 points**).