

## Quizzes for Math 222

**QUIZ 1.** Let  $f(x) = 2x + \ln x$ ,  $x \in (0, \infty)$ .

- a) Explain why  $f$  has an inverse function.
- b) Compute  $(f^{-1})'(2)$ .
- c) Find the range of  $f$ . Explain your answer.

**QUIZ 2.** a) Solve the equation

$$\ln(x + 6) + \ln(x - 3) = \ln 5 + \ln 2$$

- b) Differentiate the function  $f(x) = x^{\sin x}$ .

**QUIZ 3.** a) Compute  $\lim_{x \rightarrow \infty} \left(\frac{x}{x+1}\right)^x$ .

- b) Compute  $\int \frac{dx}{4x^2 + 1}$ .

**QUIZ 4.** Compute:

- a)  $\int e^{2x} \sin x dx$
- b)  $\int_0^{\pi/4} \tan^2 x \sec^4 x dx$ .

**QUIZ 6.** Find the area of the surface obtained by revolving the curve

$$y = \frac{e^x + e^{-x}}{2}, \quad 0 \leq x \leq \ln 2$$

about the y-axis. What is the length of this curve?

**QUIZ 7.** a) Find the solution to the differential equation

$$y' = \frac{x+1}{xy}, \quad x > 0$$

which satisfies  $y(1) = -4$ .

b) Let  $y(x)$  be the solution to  $y' = x^2 + y^2$ ,  $y(0) = 0$ . Find approximation to  $y(3)$  using Euler's method with step size 1 (i.e.  $n = 3$ ). Can you find the actual solution?

**QUIZ 8.** a) Compute  $\sum_{n=0}^{\infty} (\frac{1}{3})^{3n+2}$  or prove that it diverges.

b) A convergent sequence  $a_n$  satisfies the recurrence relation  $1 + a_n a_{n+1} = 2a_{n+2}$ . Find  $\lim_{n \rightarrow \infty} a_n$ .

c) Compute  $\lim_{n \rightarrow \infty} \sqrt[n]{n^2 + 3^n}$ .

**QUIZ 9.** a) Is the series  $\sum_{n=1}^{\infty} n e^{-n}$  convergent or divergent? Justify your answer.

b) Find all  $x$  for which the series  $\sum_{n=0}^{\infty} \frac{1+x^n}{2^n}$  converges.

**QUIZ 10.** Use appropriate test to determine whether the following infinite series converges absolutely, converges conditionally or diverges:

$$\text{a) } \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n} \quad \text{b) } \sum_{n=1}^{\infty} \frac{n!}{(2n)!} \quad \text{c) } \sum_{n=1}^{\infty} \frac{n-1}{n^n}$$

**QUIZ 11** a) Find the Taylor series expansion around  $a = 1$  of the function  $f(x) = x^4 - 4x^3 + 9x^2 - 9x + 5$ .

b) Find a power series expansion centered at 0 for  $1/(1+x)^2$ .

**QUIZ 12** Find the length of the curve  $x = e^t + e^{-t}$ ,  $y = 5 - 2t$ ,  $0 \leq t \leq 3$ . Set up, but do not evaluate, the integral which expresses the area of the surface obtained by rotating the curve about the  $x$ -axis.