Midterm Exam, Math 222,

October 21, 2008

Problem 1. (8 Points Each) Evaluate the following integrals.

a) $\int x\cos(x)\ dx$

b) $\int \tan^8(x)\ \sec^4(x)\ dx$

c) $\int e^{2x}\cos(x)dx$

d) $\int \frac{3x^2-x+1}{x(x^2+1)}\ dx$

e) $\int_1^{\sqrt{3}} \frac{1}{x^2\sqrt{4-x^2}}\ dx.$

Problem 2. (6 points) Circle the correct form for the partial fraction decomposition of the rational function $\frac{x^5-x^3+1}{(x^2-x-2)^2(x^2+x+2)}$:

a) $\frac{Ax+B}{x^2-x-2}+\frac{Cx+D}{(x^2-x-2)^2}+\frac{Ex+F}{x^2+x+2}$

b) $\frac{Ax+B}{(x^2-x-2)^2}+\frac{Cx+D}{x^2+x+2}$

c) $\frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{x+1}+\frac{D}{(x+1)^2}+\frac{Ex+F}{x^2+x+2}$

d) $\frac{A}{x-2}+\frac{C}{x+1}+\frac{Ex+F}{x^2+x+2}$

e) $\frac{A}{x^2-x-2}+\frac{B}{(x^2-x-2)^2}+\frac{C}{x^2+x+2}$

f) $\frac{A}{x-2}+\frac{B}{x+1}+\frac{Cx+D}{x^2+x+2}.$

Problem 3. (8 points) Does $\int_0^\infty \frac{2x}{1+x^4}\ dx$ converge or diverge? Why? Evaluate it if it converges.

Problem 4. (8 points) Does $\int_1^4 \frac{1}{x-2}\ dx$ converge or diverge? Why? Evaluate it if it converges.

Problem 5. (8 points) Use the Comparison Theorem to determine whether the following improper integral converges or diverges. DO NOT COMPUTE THE EXACT VALUE OF THE INTEGRAL, but show all work needed for the Comparison Theorem.

$$\int_2^\infty \frac{x}{\sqrt{x^6+4}}\ dx$$

Problem 6. (5 points each) Determine whether each sequence $\{a_n\}_{n=1}^{\infty}$ converges or diverges, and if it converges, then find its limit.

a) $a_n = \ln(2n + \sqrt{n}) - \ln(n)$

b) $a_n = \dfrac{2n^5 - n^3 + 3n}{3n^5 + 2n^2 - 3}$

c) $a_n = \dfrac{\sqrt{n}}{\ln(n)}$ for $n \geq 2$

d) $a_n = \dfrac{\tan^{-1}(n + \sqrt{n})}{\sqrt{4 + e^{-n}}}$

e) $a_n = \sqrt[n]{3^n + 1}$ (Hint: Use $3^n + 1 < 3^n + 3^n$, $\lim\limits_{n\to\infty} \sqrt[n]{2} = 1$, and the Squeeze Th.)

Problem 7. (5 points) A **convergent** sequence of **positive** numbers satisfies the recursive relation $a_n a_{n+1} = a_n + 2$. Find $\lim\limits_{n\to\infty} a_n$.